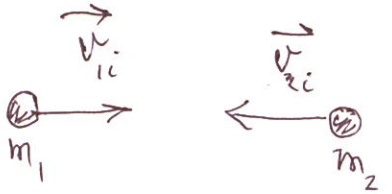
การชนแบบยืดหยุ่น

(Elastic Collision) Principle of Momentum Conservation

(i)



(f)

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{—①}$$

* ถ้าพลังงานจลน์คงที่ เรียกว่าเป็น "การชนแบบยืดหยุ่น"

$$\Sigma E_{Ki} = \Sigma E_{Kf} \longrightarrow \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{—②}$$

จะหาว่า $v_{1f} = ?$ $v_{2f} = ?$

② : $\frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2$

$$\frac{1}{2} m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = \frac{1}{2} m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) \quad \text{—③}$$

① : $m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f} - m_2 v_{2i}$

$$\frac{1}{2} m_1 (v_{1i} - v_{1f}) = \frac{1}{2} m_2 (v_{2f} - v_{2i}) \quad \text{—④}$$

③/④ :

$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \quad \text{—⑤}$$

*
$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \quad \text{—⑥}$$

$$v_{1f} = v_{2f} + v_{2i} - v_{1i} \quad \text{—⑦}$$

แทน v_{1f} จาก ⑦ ใน ① :

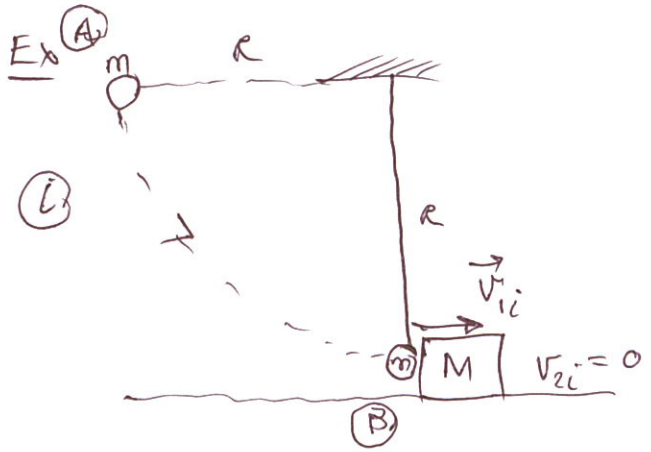
$$m_1 v_{1i} + m_2 v_{2i} = m_1 (v_{2f} + v_{2i} - v_{1i}) + m_2 v_{2f}$$

$$(m_1 + m_2) v_{2f} = 2 m_1 v_{1i} + (m_2 - m_1) v_{2i}$$

$$* \quad v_{2f} = \frac{2m_1}{(m_1+m_2)} v_{1i} + \frac{(m_2-m_1)}{(m_1+m_2)} v_{2i} \quad \text{(1)}$$

$$* \quad v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)} v_{1i} + \frac{2m_2}{(m_1+m_2)} v_{2i} \quad \text{(2)}$$

Ex (A)



$m = 4$ [kg]
$M = 6$ [kg]
$R = 20$ [m]

(A) → (B)

$$mgh_A = \frac{1}{2}mv_B^2$$

$$mgR = \frac{1}{2}mv_{1i}^2$$

$$v_{1i}^2 = 2gR$$

$$v_{1i} = \sqrt{2gR} = \sqrt{2(10)(20)}$$

$$v_{1i} = \boxed{20 \text{ [m/s]}}$$

$$v_{2i} = \boxed{0}$$

หาแบบยืดหยุ่น

$$v_{2f} = \frac{2(4)}{(4+6)}(20) + \frac{(6-4)}{(4+6)}(0)$$

$$= \boxed{16 \text{ [m/s]}}$$

$$v_{1f} = \frac{(4-6)}{(4+6)}(20) + \frac{2(6)}{(4+6)}(0)$$

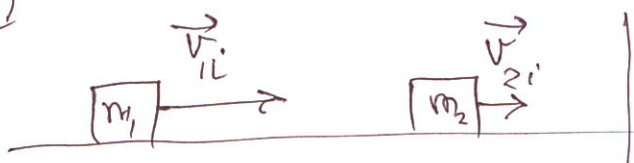
$$= \boxed{-4 \text{ [m/s]}}$$

หลังชน ลูก m เด้งกลับ ลูก M ไปทางขวา

Ex $m_1 = 1$ [kg] $\quad v_{1i} = +6$ [m/s]

$m_2 = 4$ [kg] $\quad v_{2i} = +2$ [m/s]

(i)



(f)

การชนแบบยืดหยุ่น

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2m_2}{(m_1 + m_2)} v_{2i}$$

$$= \frac{(1-4)}{(1+4)}(6) + \frac{(2)(4)}{(1+4)}(2)$$

$$= \frac{(-3)(6)}{(5)} + \frac{(8)(2)}{(5)}$$

$$= -\frac{18}{5} + \frac{16}{5} = -\frac{2}{5} \text{ [m/s]}$$

$$= \boxed{-0.4 \text{ [m/s]}}$$

$$v_{2f} = \frac{2m_1}{(m_1 + m_2)} v_{1i} + \frac{(m_2 - m_1)}{(m_1 + m_2)} v_{2i}$$

$$= \frac{(2)(1)}{(1+4)}(6) + \frac{(4-1)}{(1+4)}(2)$$

$$= \frac{(2)(6)}{5} + \frac{(3)(2)}{(5)}$$

$$= \frac{12}{5} + \frac{6}{5} = \frac{18}{5} = \boxed{3.6 \text{ [m/s]}}$$

ตรวจคำตอบ

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$6 - 2 = -(-0.4 - 3.6)$$

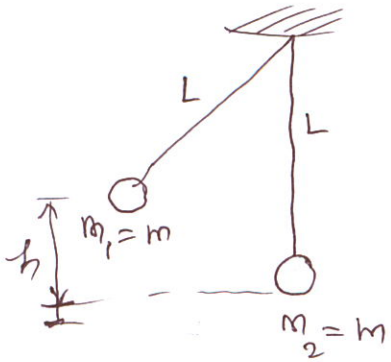
$$4 = -(-4) \quad \checkmark$$

ตรวจคำตอบ $\Sigma E_{Ki} = \Sigma E_{fi}$

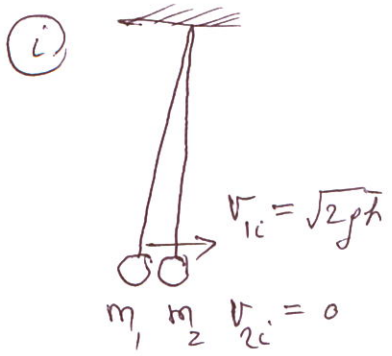
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(2)^2 = \frac{36}{2} + 8 = \boxed{26 \text{ [J]}}$$

$$\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(1)\left(-\frac{2}{5}\right)^2 + \frac{1}{2}(4)\left(\frac{18}{5}\right)^2$$

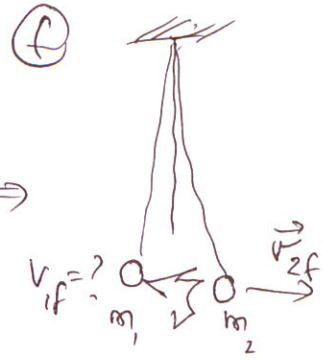
$$= \frac{2}{25} + \frac{648}{25} = \frac{650}{25} = \boxed{26 \text{ [J]}}$$



$\Rightarrow$



$\Rightarrow$



$m_1 = m_2 = m$

$$v_{2f} = \frac{2m_1}{(m_1+m_2)} v_{1i} + \frac{(m_2-m_1)}{(m_1+m_2)} v_{2i}$$

$$= \frac{2m}{2m} v_{1i} + \frac{(m-m)}{2m} v_{2i} = v_{1i}$$

$$\boxed{v_{2f} = v_{1i}}$$

ของ m_2 ขับตามเวลา m_1

$$v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)} v_{1i} + \frac{2m_2}{(m_1+m_2)} v_{2i}$$

$$= \frac{(m-m)}{2m}{}^{0} v_{1i} + \frac{2m}{2m} v_{2i} = v_{2i}$$

$$\boxed{v_{1f} = v_{2i}}$$

ตามเวลาสลับกัน

กรณี $m_2 \gg m_1$

$$v_{2f} = \frac{2m_1}{(m_1+m_2)}{}^{\approx 0} v_{1i} + \frac{(m_2-m_1)}{(m_1+m_2)}{}^{\approx 1} v_{2i}$$

$$\boxed{v_{2f} \approx v_{2i}}$$

ของ m_2 ยังคงตามเดิม

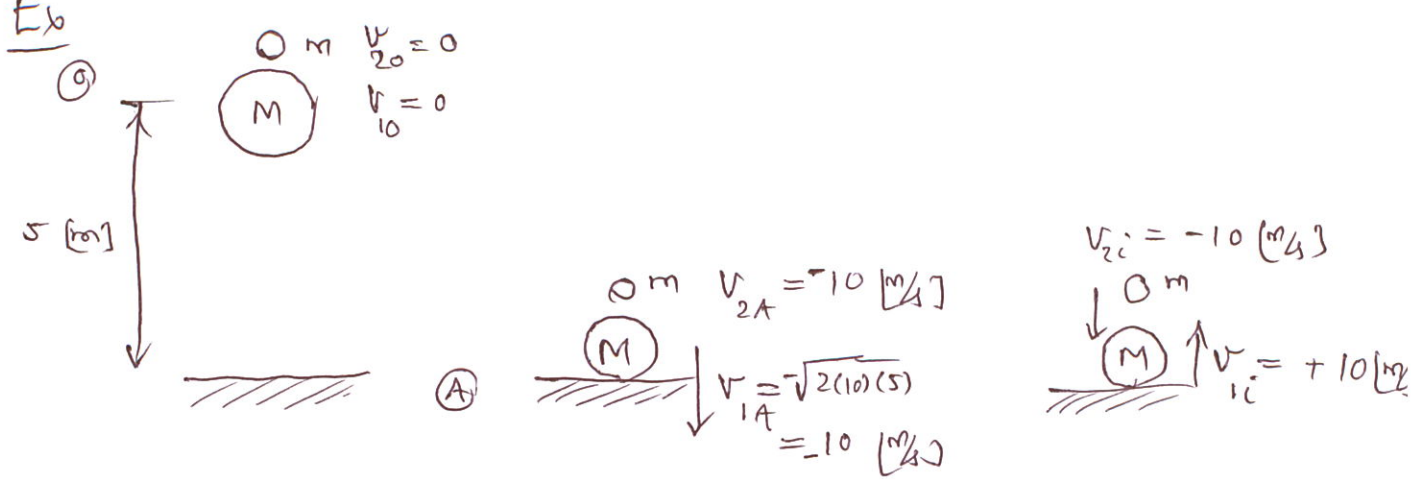
$$v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)}{}^{\approx -1} v_{1i} + \frac{2m_2}{(m_1+m_2)}{}^{\approx 2} v_{2i}$$

$$\boxed{v_{1f} = -v_{1i} + 2v_{2i}}$$

ของ m_1 ถูกชน เด้งกลับมาด้วย

$m_1 = M = 3m \qquad m_2 = m$

Eb



$$v_{2f} = \frac{2m_1}{(m_1+m_2)} v_{1i} + \frac{(m_2-m_1)}{(m_1+m_2)} v_{2i}$$

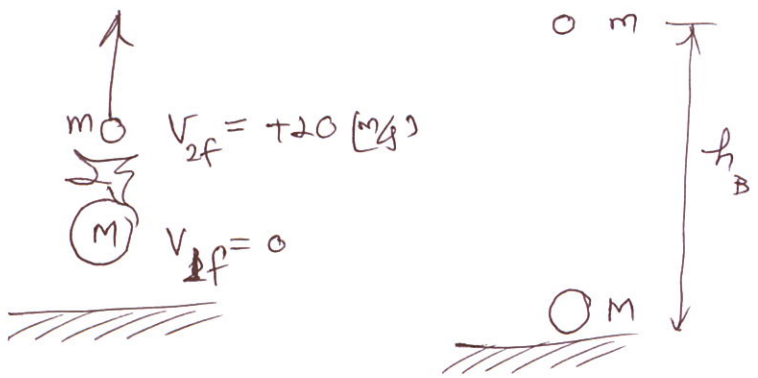
$$= \frac{2(3m)}{(3m+m)}(+10) + \frac{(m-3m)}{(3m+m)}(-10)$$

$$= \left(\frac{6}{4}\right)(+10) + \left(\frac{-2}{4}\right)(-10) = \frac{60}{4} + \frac{20}{4} = \frac{80}{4} = \boxed{20 \text{ [m/s]}}$$

$$v_{1f} = \frac{(m_1-m_2)}{(m_1+m_2)} v_{1i} + \frac{2m_2}{(m_1+m_2)} v_{2i}$$

$$= \frac{(3m-m)}{(3m+m)}(+10) + \frac{(2)(m)}{(3m+m)}(-10) = \left(\frac{2}{4}\right)(10) + \left(\frac{2}{4}\right)(-10)$$

$$= \frac{20}{4} - \frac{20}{4} = 0 = \boxed{0 \text{ [m/s]}}$$



$$\frac{1}{2} m_2 v_{2f}^2 = m_2 g h_B$$

$$h_B = \frac{v_{2f}^2}{2g} = \frac{(20)^2}{2(10)}$$

$$= \boxed{20 \text{ [m]}}$$

$\{ h_A = 5 \text{ [m]} \}$

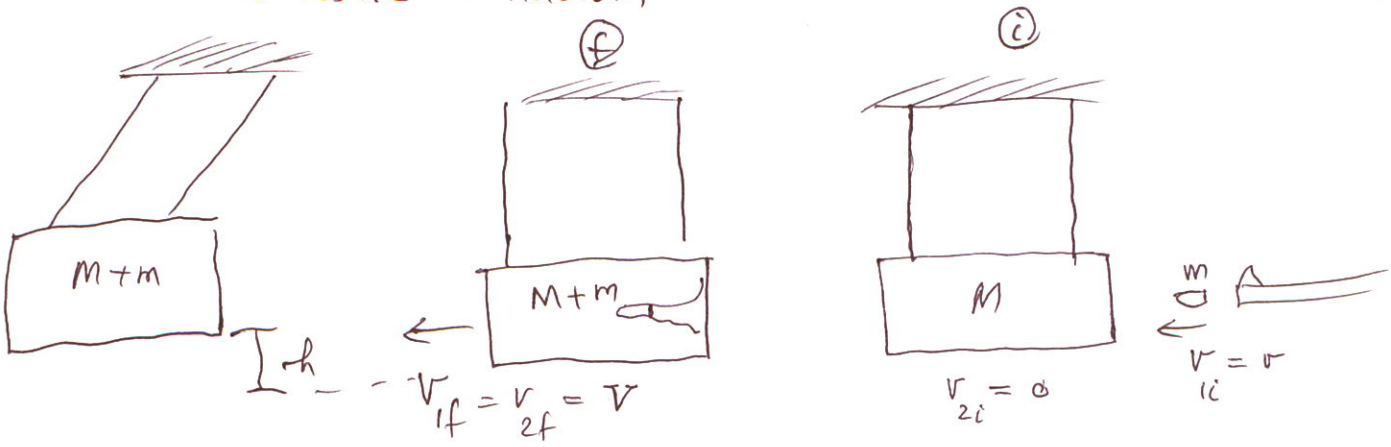
ตรวจคำตอบ $\quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$

$$(+10) - (-10) = -(0 - 20)$$

$$20 = 20 \quad \checkmark$$

Ex

Ballistic Pendulum



Inelastic collision

$m = 3$ [g]

$M = 600$ [g]

$h = 16$ [cm]

$v = ?$

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$mv + M(0) = (M+m)V$$

$$v = \frac{(M+m)}{m} V$$

$$\frac{1}{2}(m+M)V^2 = (M+m)gh$$

$$V^2 = 2gh$$

$$V = \sqrt{2gh}$$

$$v = \frac{(M+m)}{m}\sqrt{2gh}$$

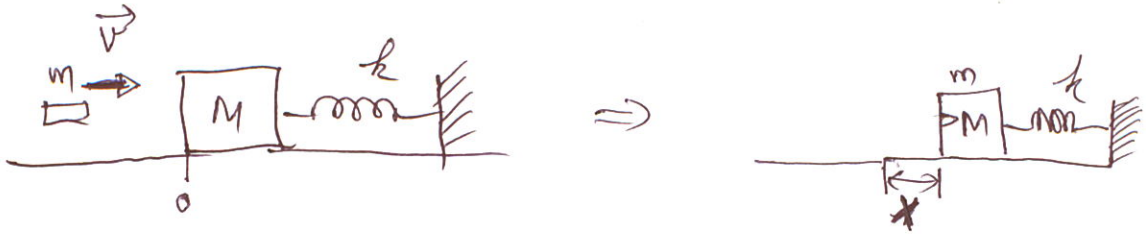
$$v = \frac{(0.6+0.003)}{0.003}\sqrt{2(10)(0.16)}$$

$$= \frac{603}{3}\sqrt{3.2}$$

$$\approx (301)(1.79)$$

$$\approx \boxed{538 \ [m/s]}$$

Ex



$m = 8$ [g]

$M = 0.992$ [kg]

$k = 400$ [N/m]

$x = 16$ [cm]

$v = ?$

$$mv = (M+m)V_f$$

$$v = \frac{(M+m)}{m} V_f$$

$$\frac{1}{2}(M+m)V_f^2 = \frac{1}{2}kx^2$$

$$V_f^2 = \frac{k}{(M+m)}x^2$$

$$V_f = \sqrt{\frac{k}{(M+m)}}\,x$$

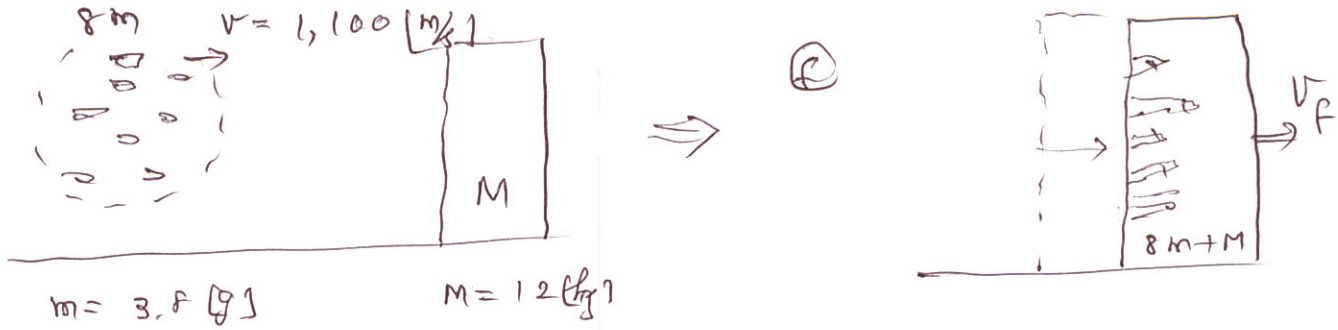
$$v = \frac{(M+m)}{m}\sqrt{\frac{k}{(M+m)}}\,x$$

$$v = \frac{(0.992+0.008)}{0.008}\sqrt{\frac{400}{(0.992+0.008)}}\,(0.16)$$

$$= \left(\frac{1}{0.008}\right)(20)(0.16) = 20 = \boxed{400 \text{ [m/s]}}$$

Ex

(i)



$m = 3.8$ [g]

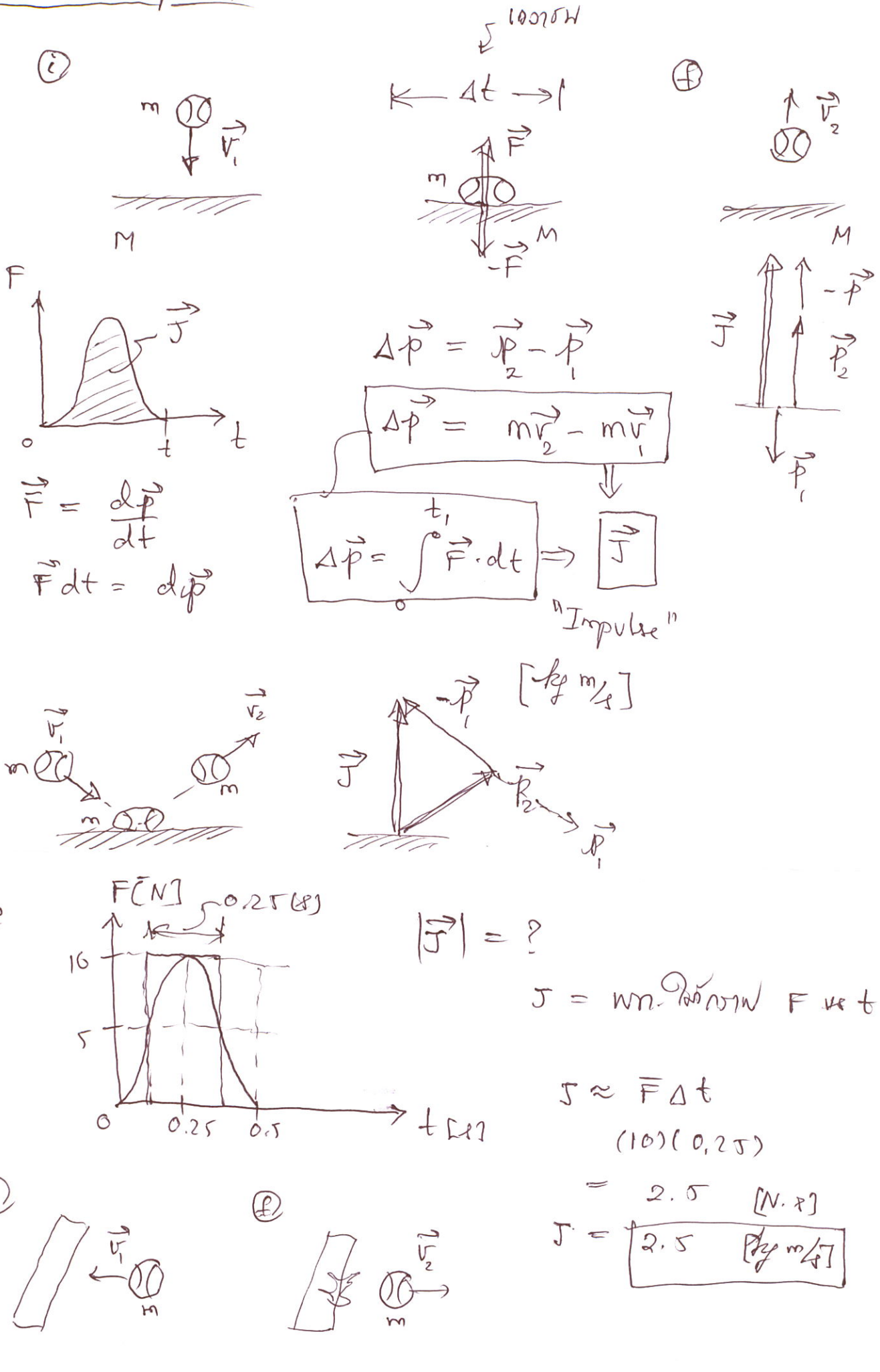
$M = 12$ [kg]

$$8mv = (8m+M)v_f$$

$$v_f = \frac{8mv}{(8m+M)} = \frac{8\times 3.8\times 10^{-3}\times 1.1\times 10^{3}}{(8\times 3.8\times 10^{-3}+12)} \approx \frac{8\times 3.8\times 1.1}{12} \approx \frac{2}{3}$$

$$= \boxed{2.8 \text{ [m/s]}}$$

การดล (Impulse)



Ex

$|\vec{J}| = ?$

J = พื้นที่ใต้กราฟ F vs t

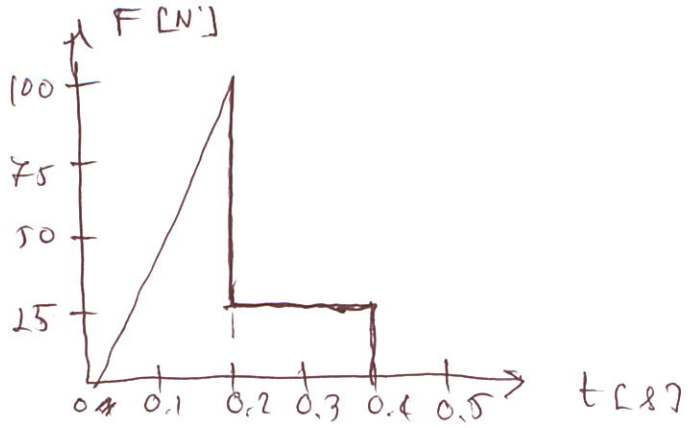
$J \approx \bar{F}\Delta t$

$(10)(0.25)$

$= 2.5$ [N.s]

$J = \boxed{2.5 \text{ [kg m/s]}}$

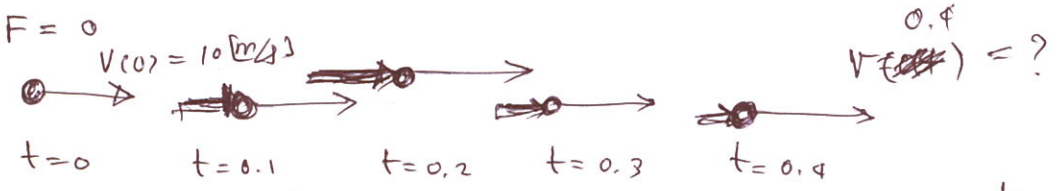
Ex



$m = 5$ [kg]

$V(0) = +10$ [m/s]

$V(0.4) = ?$



$$\Delta \vec{p} = \int_0^{t'} \vec{F}\, dt$$

$$mv_2 - mv_1 = \int_{t_1}^{t_2} F\, dt$$

$mv_2 - mv_1 = \Delta p = 15$ [kg m/s]

$mv_2 = \Delta p + mv_1$

$= 15 + (5)(10)$

$mv_2 = 65$ [kg m/s]

$v_2 = \frac{65}{5} = \boxed{13 \text{ [m/s]}}$

= พื้นที่ใต้กราฟ

$= \triangle + \square$

$= \frac{1}{2}(0.2)(100) + (0.2)(25)$

$\Delta p = 10 + 5 = \boxed{15 \text{ [kg m/s]}}$