Evaluation Models of Solvency Capital Requirement for Non-life Insurance Business

ตัวแบบการประเมินเงินทุนเพื่อความมั่นคงสำหรับธุรกิจประกันวินาศภัย

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Outline

Thesis title

Evaluation Models of Solvency Capital Requirement for Non-life Insurance Business

Questions

- **What is the Solvency Capital Requirement?**

» The amount of own funds that an insurance company is required to hold.

Question

What is Solvency II?

- New regulation framework of the European Union for insurance and reinsurance company.
- Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS)

What about in Thailand??

» Office of Insurance Commission (OIC)

Solvency II

The Solvency II is form by three pillars as follows;

Pillar 1

Quantitative Requirements

- Balance sheet evaluation
- Solvency Capital Requirement (SCR)
- Minimum Capital Requiremnet (MCR)

Pillar 2

Qualitative Requirements

Pillar 3

Disclosure

Main aspect of Solvency II

The calculation of the SCR.

Definition of SCR

Value-at-risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.

SCR formula

- Standard formula
- Internal model by setting up for own company

Literature review

Discussion on solvency capital requirement

For example,

- Eling et al. (2007) outlined the characteristics of Solvency II,
- Doff (2008) made a critical analysis of the Solvency II Proposal in the standard formula,
- Holzmuller(2009) focused on the relation between the United States risk-based capital standard, Solvency II and the Swiss Solvency test and
- Devolder (2010) studied the capital requirement under different risk measurements.

one of the most significant innovations of Solvency II

The possible use of internal models to determine the SCR.

Literature review

Many studies have focused on developing the SCR standard formula.

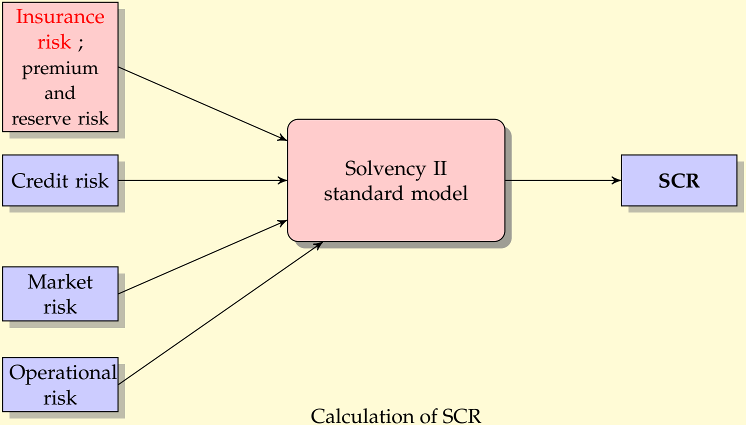
For example,

- Ohlsson, E. and Lauzeningks, J. (2009) concentrated on clarifying the one-year concept,
- Christiansen, M., Denuit, M. and Lazar, P. (2012) developed a model supporting used in Solvency II to aggregate the modular life SCR ,
- Levantesi, S. and Menzietti, M. (2012) developed a model for risk assessment in a portfolio of life annuities with long term care benefits and
- Alm, J. (2012) developed a general technique for constructing a simulation model which is able to generate the SCR.

In this thesis

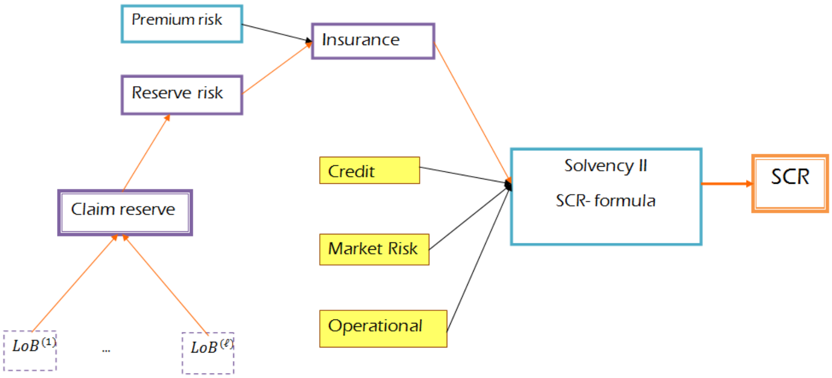
We want to develop and investigate the SCR formula.

- SCR is include both life and non-life insurance parts.
- Here, we focus on SCR for non-life insurance.



Calculation of SCR

Thesis outline



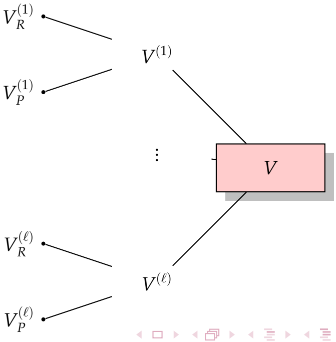
Standard formula

Standard formula

$$\mathrm{SCR}_{NL} := V \cdot g(\sigma), \tag{1}$$

where $g(\sigma) := \left(\frac{e^{N_{0.995}\sqrt{\log(\sigma^2+1)}}}{\sqrt{\sigma^2+1}} - 1\right)$

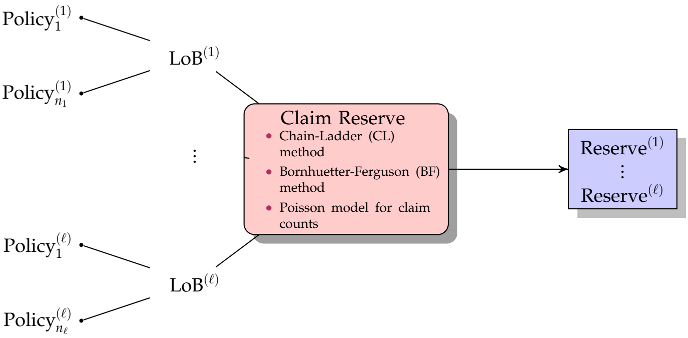
- where $\mathcal{N}_{0.995}$ is the 0.995 quantile of the standard normal distribution ($\mathcal{N}_{0.995} \approx 2.58$),
- V is a volume measure and
- σ is the combined standard deviation per volume unit of the non-lift for the lines of business (LoBs)



Computational

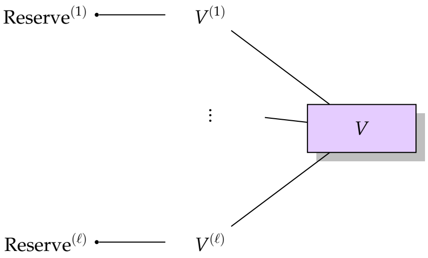
Step 1

- Calculate reserve of each line of business (LoB) in non-life insurance company.



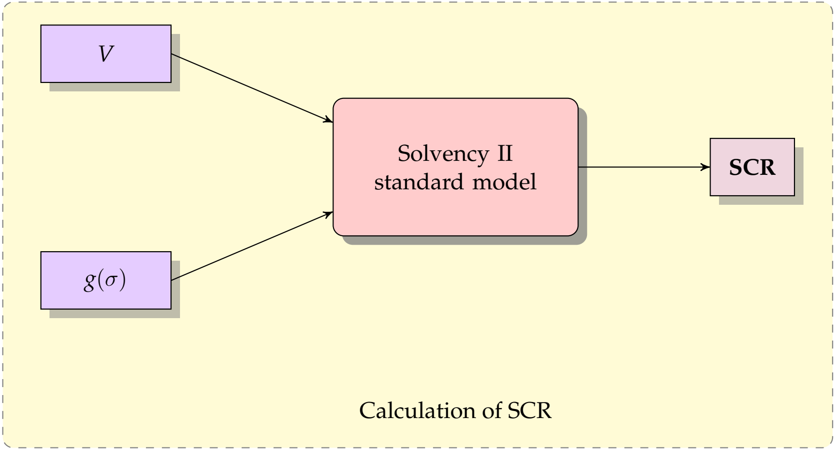
Step 2

Calculate measure V and $g(\sigma)$ to get SCR.



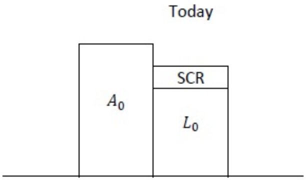
Step 2

Calculate measure V and $g(\sigma)$ to get SCR.



another SCR formula

Jonas Alm (2012) : Based on a fundamental concept to present value of future cash flows.



The company's portfolio in one year

$$A - L$$

The SCR- Based on a fundamental concept to present value of future cash flows

The regulator will conclude that

» the insurance company has enough assets to cover its liabilities, if

$$\mathrm{VaR}_{0.005}(A - L) \leq 0$$

equivalent to

$$A_0 \geq L_0 + \mathrm{VaR}_{0.005}(\Delta)$$

where Δ is the difference between assets and liabilities of insurance company changes over the coming year.

SCR formula

$$\mathrm{SCR} = \mathrm{VaR}_{0.005}(\Delta)$$

SCR formulas

SCR formula

$$\text{SCR} = \text{VaR}_{0.005}(\Delta) = \text{BE} \cdot F_U^{-1}(0.995)$$

where BE is an unbiased best estimate of the present value of the outstanding loss liability cash flow and F_U^{-1} is the quantile function of U.

Rewrite the SCR-standard formula

$$\text{SCR}_{NL} = V \cdot F_{\tilde{U}}^{-1}(0.995) \quad (2)$$

where the loss per volume $\tilde{U}$ is random variable with mean zero and variance σ^2.

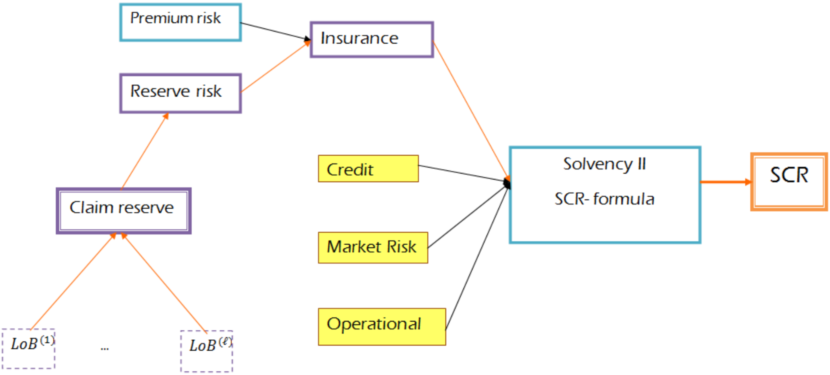
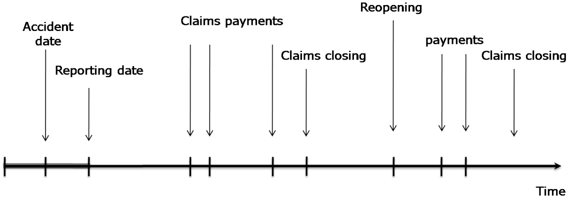


Figure : Thesis outline

Back to Claim Reserve

Why to reserve?

» Every non-life insurance company has to set up a fund for the future compensation of policyholders for claim events that have already occurred.



claims incurred

1. Reported and paid
2. Reported, not yet paid
3. Not yet reported (IBNR)

» predict the outstanding liabilities and provide a reserve

How important !!

- It is important that the claims reserve is carefully calculated,

» If it is underestimated,
  - The insurance company will not be able to fulfill its undertakings and

» If it is over estimated
  - The insurance company unnecessarily holds the excess capital instead of using it for other purposes, e.g. for investments with higher risk and, hence, potentially higher return.

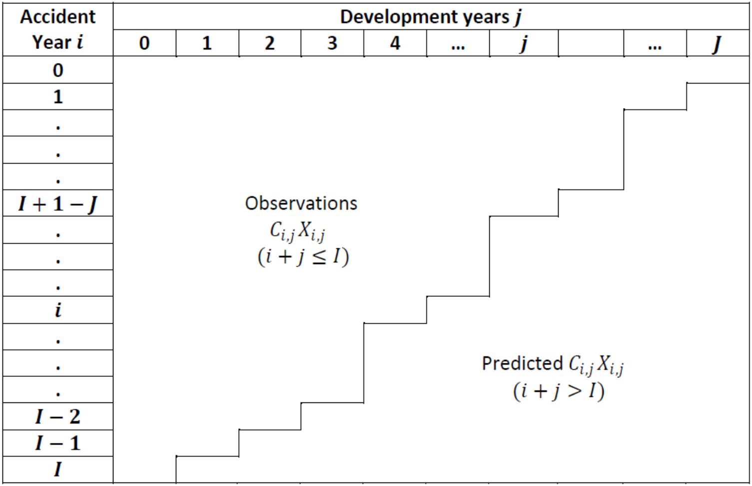
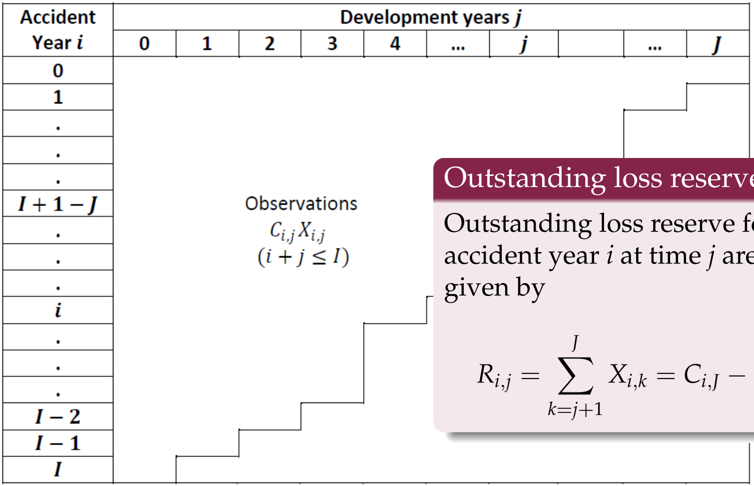


Figure : claim development



Outstanding loss reserve

Outstanding loss reserve for accident year i at time j are given by

$$R_{i,j} = \sum_{k=j+1}^{J} X_{i,k} = C_{i,J} - C_{i,j}$$

Figure : claim development

Chain-Ladder method

Why choose Chain-Ladder (CL) method

- » Most popular loss reserving technique in practice.
- » Most commonly used in the actuarial professional (also in Thailand).

How about advantage and disadvantage of CL

- Advantage
  - » it is easy.
- Disadvantages
  - » it implicitly assumes too many parameters.
  - » it does not give any idea of the variability of the reserve estimate, or a confidence interval for the reserve.

CL

Assumption 1

- Cumulative claims $C_{i,j}$ of different accident year i are independent.
- There exist development factors $f_0, ..., f_{J-1} > 0$ such that for all $0 \leq i \leq I$ and all $1 \leq j \leq J$ we have

$$E[C_{i,j}|C_{i,0}, \ldots, C_{i,j-1}] = E[C_{i,j}|C_{i,j-1}] = f_{j-1}C_{i,j-1}$$

where upper triangle in the previous figure is the observations at time I.

CL

Lemma 2

Under Model Assumption 1 we have

$$E[C_{i,J}|D_I] = E[C_{i,J}|C_{i,I-i}] = C_{i,I-i}f_{I-1}\dots f_{J-1}$$

for all $1 \leq i \leq I$.

where D_I is the upper triangular.

The outstanding claims liabilities of accident year i based on D_I

$$E[C_{i,J}|D_I] - C_{i,I-i} = C_{i,I-i}(f_{I-i}\dots f_{J-1} - 1)$$

The CL factors f_j, for $j = 0, \dots, J-1$

$$\widehat{f_j} = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}} \tag{3}$$

CL

CL estimator

The CL estimator for $E[C_{i,j}|D_I]$ is given by

$$\widehat{C_{i,j}}^{CL} = \widehat{E}[C_{i,j}|D_I] = C_{i,I-i}\widehat{f}_{I-i}\ldots\widehat{f}_{j-1} \tag{4}$$

for $i + j > I$.

» The algorithm that leads to the CL reserves can be derived from this equation.

another method for claim reserve

- another method for claim reserve

» Support vector machine (SVM)
- Machine learning is about learning structure from data.
- Received a lot of attention in machine-learning community.

» Basic idea of the Support Vector Regression (SVR)
- in case ε-Support Vector Regression (ε-SVR).

Support Vector Machine (SVM)

- Let D be the training set, $D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l)\}$, where $x_i \in \mathbb{R}^d$ are input data and $y_i \in \mathbb{R}$ are the target values.
- The SVR method first maps the data $x \in \mathbb{R}^d$ into some a prior Hilbert space $\mathcal{F}$, called the **feature space**, via a nonlinear function $\Phi : \mathbb{R}^d \to \mathcal{F}$.

In this feature space,

The prediction function

$$f(x) = \langle w, \Phi(x) \rangle + b \tag{5}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{F}$, $w \in \mathcal{F}$ and $b \in \mathbb{R}$.

Support Vector Machine (SVM)

In the ε-SVR method, the problem of learning is to find the best function:

$$R(w,b) = \frac{1}{l}\sum_{i=1}^{l} |y_i - f(x)|_\varepsilon + \frac{1}{2}\langle w, w\rangle$$

where

$$|a|_\varepsilon = \begin{cases} 0, & \text{if } |a| < \varepsilon \\ |a| - \varepsilon & \text{otherwise} \end{cases}$$

The solution $f(x)$ of this ε-SVR problem minimizes the deviation from $|f(x_i) - y_i|$ for $i = 1, \ldots, l$.

Support Vector Machine (SVM)

In the ε-SVR learning method, the values of w and b are determined by ;

$$\text{Minimize}_{w,b} \frac{1}{2}\|w\|^2 + P \cdot \sum_{i=1}^{l} (\xi_i + \hat{\xi}_i) \tag{6}$$

subject to:

$$\begin{aligned} &\xi_i, \hat{\xi}_i \geq 0 && i = 1, \ldots, l \\ &(\langle w, \Phi(x_i)\rangle + b) - y_i \leq \varepsilon + \xi_i && i = 1, \ldots, l \\ &y_i - (\langle w, \Phi(x_i)\rangle + b) \leq \varepsilon + \hat{\xi}_i && i = 1, \ldots, l \end{aligned}$$

- where P is a constant parameter that penalizes the ε-insensitive errors.
- The error occurring if $f(x_i)$ is above (below) y_i are represented by the ξ_i ($\hat{\xi}_i$) slack variables.

Support Vector Machine (SVM)

Its dual form by using Lagrange multipliers $\alpha_i, \hat{\alpha}_i$:

$$\text{Maximize}_{\alpha_i,\hat{\alpha}_i} \sum_{i=1}^{l} (\hat{\alpha}_i - \alpha_i) y_i - \varepsilon \sum_{i=1}^{l} (\hat{\alpha}_i + \alpha_i) \quad (7)$$

$$- \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} (\hat{\alpha}_i - \alpha_i)(\hat{\alpha}_j - \alpha_j) \langle \Phi(x_i), \Phi(x_j) \rangle$$

subject to:

$$\sum_{i=1}^{l} (\hat{\alpha}_i - \alpha_i) = 0$$

$$0 \le \alpha_i, \hat{\alpha}_i \le P$$

- Let $\alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_l^*)$ and $\hat{\alpha} = (\hat{\alpha}_1^*, \hat{\alpha}_2^*), \ldots, \hat{\alpha}_l^*$ denote the optimal solution of the dual problem.

Support Vector Machine (SVM)

The complementary Karush Kuhn-Tucker (KKT) conditions for this dual problem at the optimal solution are

$$
\begin{aligned}
w - \sum_{i=1}^{l} (\hat{\alpha}_i - \alpha_i)\Phi(x_i) &= 0 \qquad (8)\\
\text{for } i &= 1, \ldots, l\\
\alpha_i^* (\langle w, \Phi(x_i)\rangle + b - y_i - \varepsilon - \xi_i) &= 0\\
\hat{\alpha}_i^* (y_i - \langle w, \Phi(x_i)\rangle + b - \varepsilon - \hat{\xi}_i) &= 0\\
\hat{\alpha}_i^* \cdot \alpha_i^* = 0, \hat{\xi}_i, \xi_i &= 0\\
(\hat{\alpha}_i^* - P)\hat{\xi}_i = 0, (\alpha_i^* - P)\xi_i &= 0
\end{aligned}
$$

Support Vector Machine (SVM)

Then equation (5) can be rewritten as

The prediction function

$$f(x) = \sum_{i=1}^{l} (\hat{\alpha}_i - \alpha_i)\langle \Phi(x_i), \Phi(x) \rangle + b^* \tag{9}$$

where b^* is chosen so that $f(x_i) - y_i = -\varepsilon$ for any i such that $\alpha_i^* \in (0, P/l)$.

» Before training data, we construct the data from the observation information (upper triangular) first.

Thesis outline

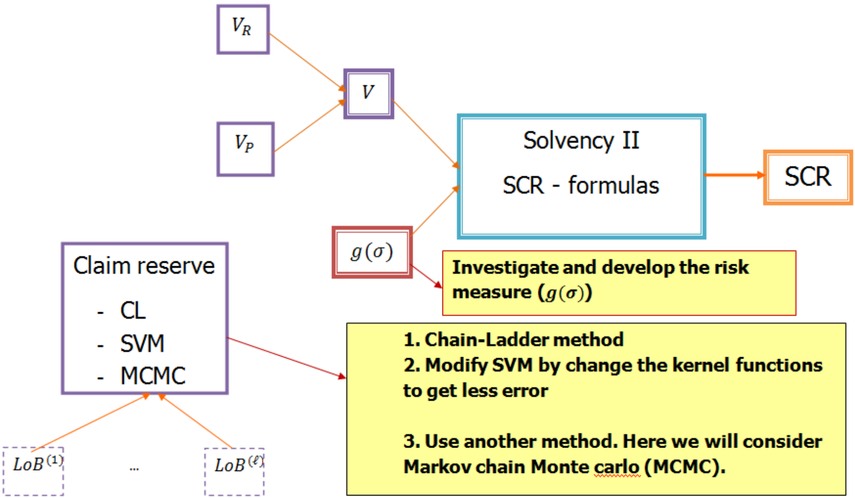


Figure : thesis outline

Research Objectives

With the main aspect of Solvency II, calculation of the SCR, our research will focus as follows:

1. **Loss reserve**
   - To estimate the claim reserve by using CL method and SVM and modify SVM for claim reserve.
   - To compare the loss reserve between CL method and SVM.
2. **The SCR evaluation**
   - To calculate the SCR standard formula and the SCR formula based on a fundamental concept to present value of future cash flows.
   - To develop the SCR standard formula by a new issue.
3. To compare these SCR formulas; the SCR standard formula and the SCR formula based on a fundamental concept to present value of future cash flows and the SCR standard formula by a new issue.

Scope and Limitations of the study

Loss reserve

1. Claims arrivals happen at time T_i. The claim arrival process (T_i) constitutes those the claims arrival satisfying $0 \leq T_1 \leq T_2 \leq ....$
2. Let X_i be the claim size of the ith claim arriving at time T_i. The process (X_i) forms an independent identically distribution(i.i.d.) sequence of non-negative random variables. Assume that the process (X_i) and (T_i) are *mutually independent*.
3. The claim number process is the number of the claims which occurred by time t :

$$N_t = \#\{i \geq 1 : T_i \leq t\}, t \geq 0.$$

   Note that $N = (N_t)_{t \geq 0}$ is a counting process on $[0, \infty]$.
4. We will do the analysis with the incremental claims, obtained by differencing successive cumulative amounts.

Scope and Limitations of the study

The SCR evaluation

The insurance types studied in this paper relate to the lines of (LoBs) defined in the Solvency II framework. The following shows the formal definition of three types of LoBs;

- **Accident**
- **Sickness**
- **Motor**

Research procedure

The research procedure of this thesis are the following:

Loss reserve

(1)

- Estimate the LoBs reserves by using CL.
- Modify SVM and estimate the LoBs reserves by using SVM.

The SCR evaluation

(2)

- Computing the SCR by using the standard SCR of solvency II.
- Computing the SCR by using the SCR formula based on a fundamental concept to present value of future cash flows.
- Investigation and computing the SCR formula in a new issue.

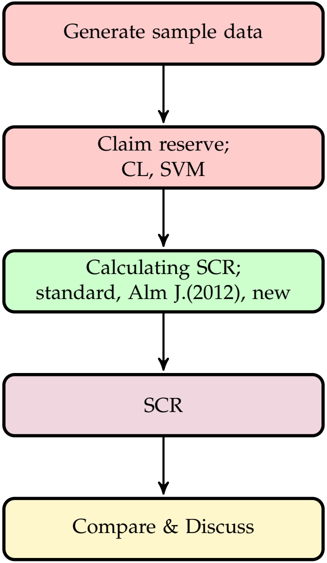
Research procedure

Comparison and discussion

(3)

- Compare the loss reserves between the results using CL method and SVM.
- Compare and discuss the SCR values which are evaluated by the standard SCR of solvency II ,the SCR formula based on a fundamental concept to present value of future cash flows and the new SCR.

Research procedure



Expected Results

1. The SCR values, using the standard model of solvency II and the SCR formula based on a fundamental concept to present value of future cash flows, are evaluated.
2. A new model of SCR is developed and evaluated.

Year	2012		2013		2014
Activities	Apr-May	Jun-Dec	Jan-May	Jun-Dec	Jan-Apr
1. Literature survey and study the non-life insurance mathematics.	$\leftrightarrow$	$\leftrightarrow$			
2. Criticism and possible extensions of Solvency II.		$\leftrightarrow$	$\leftrightarrow$		
3. Investigation of the claims reserving method.		$\leftrightarrow$	$\leftrightarrow$		
4. Investigation of the solvency capital requirement(SCR).			$\leftrightarrow$	$\leftrightarrow$	
5. Investigation the applicability of 1, 2 and 3 into the non-life insurance company in Thailand.				$\leftrightarrow$	
6. Thesis preparation.				$\leftrightarrow$	$\leftrightarrow$

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I would welcome any comments and suggestions.

Thank you for your attention.