FORECASTING IN FINANCIAL MARKET USING MARKOV REGIME SWITCHING AND PRINCIPAL COMPONENT ANALYSIS.

NOP SOPIPAN

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In Thai

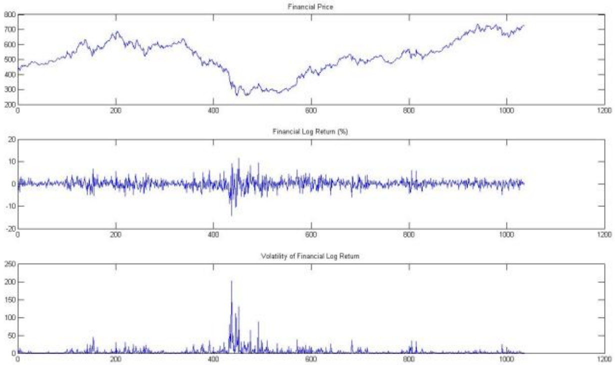
การพยากรณ์ในตลาดทางการเงิน ด้วย
วิธีการวิเคราะห์องค์ประกอบหลัก และ
วิธีสับเปลี่ยนสถานะมาร์คอฟ

The Beginning

Prof.Dr.Pairote Sattayatham

Prof.Dr.Bhusana Premanode

Characteristics of returns



Property in Financial returns

- (Weakly) Stationary process.
- Look liked White noise process
- The model assume random walk model.

Forecast Financial Return

Let $\{P_t\}$ denote the series of the financial price at time t and $\{r_t\}_{t>0}$ be the logarithmic return (in percent) on the financial price at time t be a sequence of random variables on a probability space$(\Omega, \mathbf{F}, \mathbf{P})$, i.e.

$$r_t = 100 \cdot ln(\frac{P_t}{P_{t-1}}) \quad (1)$$

Let r_t be explain in mean equation, i.e.

$$r_t = \mu_t + \varepsilon_t \quad (2)$$

with mean and variance:

$$\mu_t = E(r_t|F_{t-1}), h_t = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}]$$

Forecasted inFinancial Return

- Mehmet A.(2008), Marcucci J. (2005) assume mean equation in returns is constant
- Easton and Faff (1994), and Kyimaz and Berument (2001) assume mean equation in returns with a one week delay into the regression model
- Supot C.(2003)assume mean equation in returns with Autoregressive process.

Construct Model

- The constant mean equation cannot be forecast accurately due to the inaccuracy of the financial data, since the financial returns depend concurrently and dynamically on many economic and financial variables.
- The fact that the return has a statistically significant autocorrelation indicates that lagged returns might be useful in predicting future returns.
- We consider add some explanatory variables and stationary Autoregressive Moving-average order p and q (ARMA (p,q)). Then we add them into a mean equation in order to increase accuracy.

Construct Model

Research Objectives

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3. Construct heteroskedasticity model for forecasting of volatility with GARCH-type model and Markov Regime Switching GARCH model.
4. Compare and evaluate the performance of models between constant mean equation and model of equation (3) with loss functions.

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The process of research had been following :

- 1^{st}**Paper**.N. Sopipan, P. Sattayatham and B. Premanode , *"Forecasting Volatility of Gold Price using Markov Regime Switching and Trading Strategy"*, Journal of mathematical finance.Vol. 2(1), pp 121-131 , 2012.

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- **Next Step** Use PCA to solve multicollinearity problem in explanatory varibles in mean equation forecast return.

Scope and limitations

- The forecasting in Markov Regime Switching for simplicity, this thesis assume presence of two regimes and order of the GARCH-type is (1,1).

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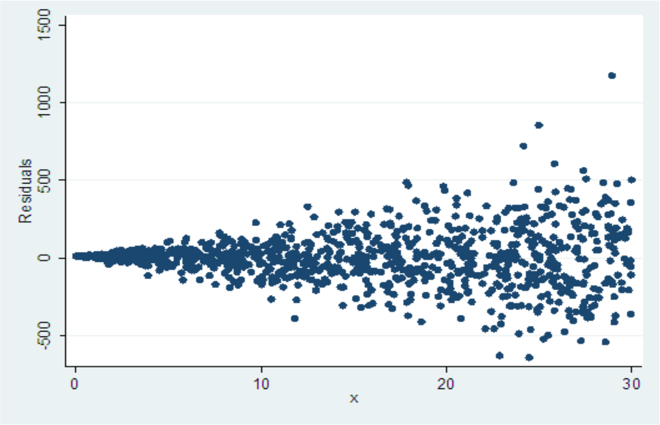
- The standardized errors follow student-t, generalized error distributions (GED) and normal distribution.

Heteroskedasticity



Heteroskedasticity

The generalized autoregressive conditional heteroskedasticity (GARCH)-type models mainly capture for problem of heteroskedasticity.

- GARCH model
- The Exponential GARCH (EGARCH) model
- GJR-GARCH model

Heteroskedasticity model

- **GARCH(1,1)**:[Bollerslev (1986)]

$$\varepsilon_t = \eta_t \sqrt{h_t}, h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

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- **EGARCH(1,1)**:[Nelson (1991)]

$$ln(h_t) = \alpha_0 + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \beta_1 ln(h_{t-1}) + \xi \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \quad (4)$$

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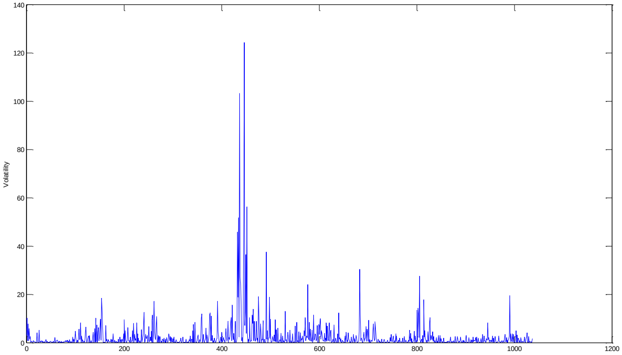
- **GJR-GARCH(1,1)**:[Glosten, Jagannathan and Runkle (1993)]

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 (1 - I_{\{\varepsilon_{t-1} > 0\}}) + \beta_1 h_{t-1} + \xi \epsilon_{t-1}^2 (I_{\{\varepsilon_{t-1} > 0\}}) \quad (5)$$

Spurious high persistence problem



Review of Markov Regime Swithcing

- Hamilton (1989,1990) extended MRS for analyzing structural breaks in financial return.
- Hamilton and Susmel (1994) and Cai (1994) proposed MRS ARCH.
- Gray (1996)proposed MRS GARCH.
- Klassen (2002) modification of Gray's MRS GARCH.

Concept of Markov Regime Swithcing

- Generalization of the simple dummy variables approach.
- Allow regimes (called states) to occur several periods over time.
- In each period t the state is denoted by S_t.
- There can be N possible states: $S_t = 1, \ldots, N$.

Markov Regime Swithcing

- Probability S_t equals some particular value j depends on the past only through the most recent value S_{t-1}: $P\{S_t = j | S_{t-1} = i, S_{t-2} = k, \ldots\} = P\{S_t = j | S_{t-1} = i\} = p_{ij}$.

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- Process is described as an N-state Markov Chain with transition probabilities $\{p_{ij}\}_{i,j=1,2,\ldots,N}$ and $p_{i1} + p_{i2} + \ldots + p_{iN} = 1$

Markov Regime Swithcing

- The conditional probability density function for the observations r_t given the state variables S_t, S_{t-1} and F_{t-1} is $f(r_t|S_t, S_{t-1}, F_{t-1})$

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- The chain rule for conditional probabilities yields then for the joint probability density function for the variables r_t, S_t, S_{t-1} given F_{t-1} is
  $f(r_t, S_t, S_{t-1}|F_{t-1}) = f(r_t|S_t, S_{t-1}, F_{t-1})Pr(S_t, S_{t-1}|F_{t-1})$

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- The log-likelihood function to be maximized with respect to the unknown parameters becomes $l(\theta) = \sum_{t=1}^{T}[log(\sum_{S_t=1}^{N}\sum_{S_{t-1}=1}^{N})f(r_t, S_t, S_{t-1}|F_{t-1})]$

Markov Regime Swithcing GARCH models

The MRS-GARCH model with only two regimes can be represented as follows.

$$r_t = \mu_{t,S_t} + \varepsilon_t = \mu_{t,S_t} + \eta_t \sqrt{h_{t,S_t}} \tag{6}$$

$$h_{t,S_t} = \alpha_{0,S_t} + \alpha_{1,S_t}\varepsilon_{t-1}^2 + \beta_{1,S_t} h_{t-1} \tag{7}$$

where $S_t = 1, 2$, μ_{t,S_t} is the mean and h_{t,S_t} is the volatility under regime S_t on F_{t-1} .

Multicollinearily

Multicollinearity is a statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated.

Multicollinearily

For the mean equation in (3) i.e.

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t, \\ \mu_t &= \phi_0 + \sum_{i=1}^{k} \beta_i X_{it} + \sum_{i=1}^{p} \phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \end{aligned}$$

Multicollinearity, or high correlation between the explanatory variables in a regression equation. One method for removing such multicollinearity and redundant independent variables is **principal component analysis (PCA)**.

Principal Component Analysis

The idea of PCA is to find linear combinations α_i such that Z_i and Z_j are uncorrelated for $i \neq j$ and the variances of Z_i are as large as possible. More specifically:

- The first principal component of X is the linear combination $Z_1 = \alpha_1^{'} X$ that maximizes Var (Z_1) subject to the constraint $\alpha_1^{'} \alpha_1 = 1$.
- The second principal component of X is the linear combination $Z_2 = \alpha_2^{'} X$ that maximizes Var (Z_2) subject to the constraint $\alpha_2^{'} \alpha_2 = 1$ and $\text{Cov}(Z_2, Z_1) = 0$.

Principal Component Analysis (Cont.)

- The kth principal component of X is the linear combination $Z_k = \alpha_k^{'} X$ that maximizes Var (Z_k) subject to the constraint $\alpha_k^{'}\alpha_k = 1$ and $\text{Cov}(Z_k, Z_{k'}) = 0$ for $k \neq k'$

Principal Component Analysis (Cont.)

The new variables from the PCA become ideal to use as predictors in a regression equation since they optimize spatial patterns and remove possible complications caused by multicollinearity.

Model of returns

The model of returns (2) as follow:

$$r_t = \mu_t + \varepsilon_t$$

This thesis use three mean equations of return as follow:
(1) Mean equation are constants.

$$\mu_t = \delta.$$

$$\mu_t = \delta_{S(t)}.$$

Model of returns

(2) Mean equation as stationary ARMA(p,q) includes day of the week effect.

$$\mu_t = \Sigma_{i=1}^{5} \beta_i D_{it} + \Sigma_{j=1}^{p} \phi_j r_{t-j} - \Sigma_{k=1}^{q} \theta_k \epsilon_{t-k},$$

where
$D_{it}, i = 1, ..., 5$ are dummy variables which take on the value of 1 if the corresponding return of the five days respectively and 0 otherwise,
$\beta_i, i = 1, ..., 5$ are coefficients which represent the average return for each day of the week ,
$\phi_j, \theta_k, i = 1, ..., p, k = 1, ..., q,$ are coefficients which represent the ARMA (p, q).

THANK YOU FOR ATTENTION