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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต
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REAL OPTION ANALYSIS ON RENEWABLE ENERGY POLICY FOR ETHANOL PLANTS IN CHINA

Hui Zhao

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Applied Mathematics

Suranaree University of Technology

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REAL OPTION ANALYSIS ON RENEWABLE ENERGY POLICY FOR ETHANOL PLANTS IN CHINA

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วิทยานิพนธ์นี้จะศึกษาผลกระทบใน นโยบายพลังงานทดแทนเกี่ยวกับโรงงานเอทานอลจากเซลลูโลสในประเทศจีนโดยใช้การวิเคราะห์ด้วยเรียลออปชั่น สมมุติว่ามีการแบ่งการก่อสร้างออกเป็นสองขั้นและมีตัวแปรสุ่มหลักสองตัวคือ ทัศนะของรัฐบาล และทัศนะของผู้ประกอบการ ในลำดับแรกวิทยานิพนธ์นี้ จะขยายขอบเขตของการวิจัยเกี่ยวกับการวิเคราะห์ออปชันเชิงจริงให้รวมถึงโครงการพลังงานทดแทน โดยเฉพาะอย่างยิ่ง การลงทุนพลังงานเอทานอล ในลำดับสอง วิทยานิพนธ์นี้จะสร้างตัวแบบชนิดดิสครีตด้วยวิธีแลตทิส ในลำดับสาม วิทยานิพนธ์นี้จะสร้างตัวแบบต่อเนื่องเพื่อวิเคราะห์ปัญหาด้วยวิธีกำหนดการพลวัต ในท้ายที่สุด หลังจากที่ได้แสดงการวิเคราะห์ผลกระทบจากนโยบายให้เงินอุดหนุนแก่ผู้ผลิต และสัดส่วนการการลงทุนในขั้นที่หนึ่งแล้ว วิทยานิพนธ์นี้ได้พบว่าการปรับปรุงเทคโนโลยีและการใช้ประโยชน์อย่างเต็มที่จากวัตถุดิบจะเป็นวิธีที่มีประสิทธิภาพในการเพิ่มรายได้ของโรงงานเอทานอลจากเซลลูโลส

สาขาวิชาคณิตศาสตร์ ลายมือชื่อนักศึกษา Hui Zhao

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REAL OPTION ANALYSIS / RENEWABLE ENERGY POLICY / CELLULOSIC ETHANOL / LATTICE TREE METHOD / DYNAMIC PROGRAMMING

This thesis investigates the impact of renewable energy policy for cellulosic ethanol plants in China with two construction stages and double stochastic variables under government and investors perspectives based on real option analysis. Firstly, the thesis generalizes the research about the real option analysis to include renewable energy projects, especially fuel ethanol investment. Secondly, the thesis constructs a discrete model using the lattice tree method. Thirdly, the thesis constructs a continuous model by the dynamic programming approach. Finally, after showing the basic analysis of the influence of the subsidy policy and the proportion of the first construction stage cost, the thesis finds that improving the technology and making full use of the raw materials are an effective way to increase the revenues of the cellulosic ethanol plants.

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Hui Zhao

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LIST OF ABBREVIATIONS

EPI	Earth Policy Institute
EIA	Energy Information Administration
RFA	Renewable Fuels Association
NDRC	National Development and Reform Commission of China
NPV	Net Present Value
DCF	Discounted Cash Flow
PV	Present Value
ROA	Real Option Analysis
RO	Real Option
GBM	Geometric Brownian Motion
CRR	Cox-Ross-Rubinstein
BM	Brownian Motion
B-S	Black-Scholes
SDE	Stochastic Differential Equation
PDE	Partial Differential Equation
ODE	Ordinary Differential Equation

CHAPTER I

INTRODUCTION

1.1 Background

In recent decades, as the energy crisis and environmental issues have become increasingly severe, most countries in the world have been vigorously developing renewable energy projects. As a kind of clean and reproducible resource, fuel ethanol can replace part of the gasoline demand, so it has become one of the focuses of attention in the renewable energy field.

According to the historical data from energy institutions such as the Earth Policy Institute (EPI), the Energy Information Administration (EIA) and the Renewable Fuels Association (RFA), the fuel ethanol industry of the world started from 1970s and ushered in the development spurt in the 21st century as shown in Figure 1.1. In 2011, the total production of fuel ethanol in the world had already achieved 22,742 millions of gallons, which was close to the 6 times the output in 2001. Three years later, the output had increased to 24,570 millions of gallons in 2014.

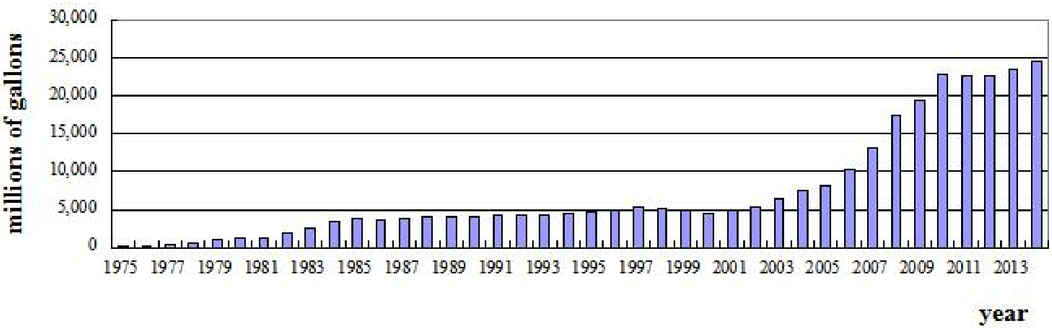


Figure 1.1 The fuel ethanol production in the world.

As a big country of energy production and consumption in the world, the energy demand of China has increased greatly with the rapid economic development of the society and the economy. In fact, China has the ability and strength to develop renewable energy and has been committed to the related research. The Chinese fuel ethanol industry based on grain, such as corn or wheat, began in 2001. However, the government gradually closed out the grain-based ethanol projects from 2006. It was evident from the reports of National Development and Reform Commission of China (NDRC) that stated the development principles of the renewable energy industry: It cannot compete with human food and land for food, it cannot destroy the ecology. Therefore, the Chinese government began to pay attention to the 2nd generation non-grain bio-fuel projects, especially the cellulosic ethanol project. In 2007, the Middle and Long Term Program of Renewable Energy Development of China stated that the available non-grain ethanol should reach more than 10 million tons in 2020. Under the guidance of these above principles, the Chinese government strives to develop the cellulosic ethanol which has been based on corn cob or corn stalk in recent years. SHANDONG LONGLIVE BIO-TECHNOLOGY CO., LTD is the first large-scale cellulosic ethanol producer with annual capacity of 50,000 tons, which had been put into operation in Shandong province in 2012. Through a special technology, the producer can use corn cob as the main raw material to produce ethanol, xylitol and other high value products at the same time as shown in Table 1.1 and Figure 1.2.

Table 1.1 The raw material and products in the cellulosic ethanol project. (Kang, 2014)

Raw Material	Corn Cob (10 tons)
Products	Pure Lignin (1.0 ton)
	Xylitol (1.2 tons)
	Ethanol (1.5 tons)

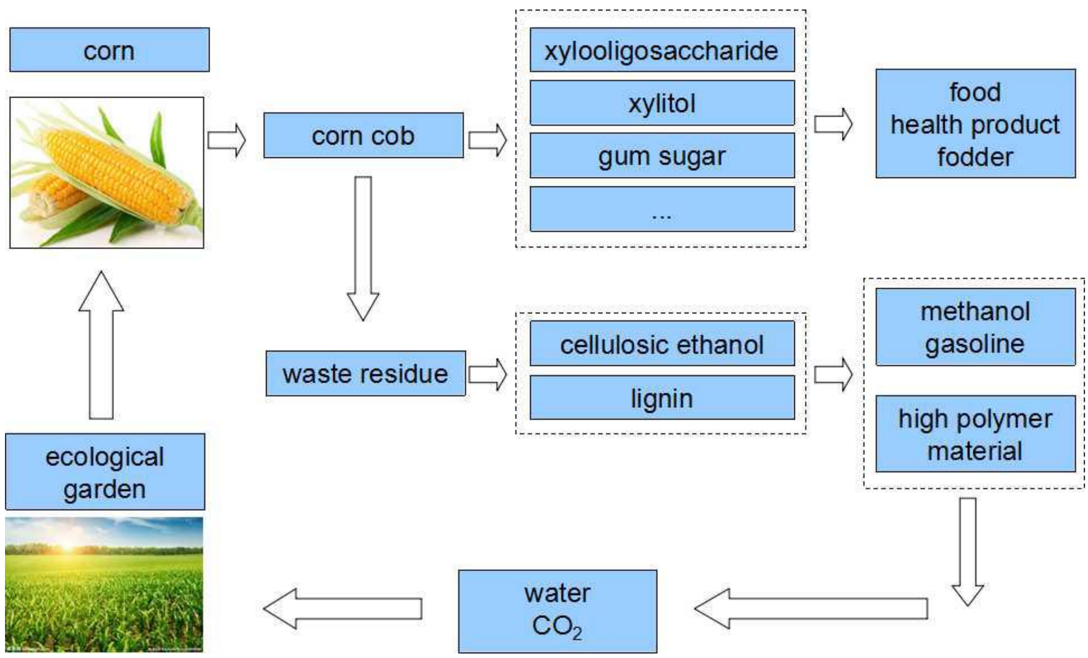


Figure 1.2 Flow diagram of the 2nd generation cellulosic ethanol process.

Because of the high research and development cost, long planning process, high investment risks and other uncertainties, the promotion of each kind renewable energy is inseparable from the support by government policies and statutes. In order to encourage the investors, many countries have made special policies for the fuel ethanol industry. For example, subsides and tax incentives are adopted widely and commonly in the world. Similarly, the Chinese government has also made a subsidy plan for the cellulosic ethanol industry - the producer can obtain the subsidy of 800 Chinese yuan from the government for every ton cellulosic ethanol produced. There is, however, a significant difference with other countries: The cellulosic ethanol project and the price of fuel ethanol are controlled by the Chinese government, so that the subsidy policy is a sensitive issue for both government and investors.

1.2 Research Objectives

Up to now, few scholars have focused on the real option application to fuel ethanol projects. Some studies (Lee et al., 2010; Sharma et al., 2013; Zhang et al., 2014) investigated the investment benefits of renewable energy policy based on the binomial lattice tree model. Especially, Zhang et al. (2014) used the lattice tree model and indicated the benefits of the renewable energy policy under two perspectives - government and investors. Meanwhile, some researches chose the real option approach developed by Dixit and Pindyck (1994), such as dynamic programming and contingent claim analysis. For example, Schmit, Luo and Tauer (2009) discussed the decisions of investment and operation for the corn-based dry-grind ethanol plants. Kirby and Davison (2010) set up a real option model to analyze the valuation of an ethanol plant as a spark spread between the corn price and the gasoline price. Schmit, Luo and Conrad (2011) estimated the influence of U.S. ethanol policy on the plant investment decisions with two stochastic variables. Maxwell and Davison (2014) quantified the impact of an abrupt change in the government policy of corn ethanol facilities.

Hence, the objective of the thesis is to evaluate the influence of renewable energy policy to cellulosic ethanol plants in China. In order to investigate the effect of the renewable energy policy and observe the regularity with the proportion of the stage-1 construction cost under government and investor perspectives, this thesis establishes two different real option models with two construction stages and double stochastic variables. The first model is based on the lattice tree method introduced by Guthrie (2009). The second one uses the dynamic programming approach developed by Dixit and Pindyck (1994). These two stochastic variables are independent in the first model, but dependent in the second one.

Although the lattice tree and dynamic programming approaches are commonly used in real option analysis, few people use them to construct real option models with multistage and multivariate characteristics at the same time. The thesis will do some work towards this direction.

1.3 Outlines of the Thesis

The remainder of this thesis is organized as follows: Chapter II provides a literature review about the traditional method and real option analysis. Meanwhile, it generalizes the application of the real option analysis to renewable energy, especially the fuel ethanol project. Chapter III establishes a real option model with two construction stages and double stochastic variables based on the lattice tree method, and discusses the basic analysis. With the same basic assumptions about the numbers of construction stages and stochastic variables, Chapter IV presents a real option model based on the dynamic programming approach, and shows the basic conclusions as well. Finally, Chapter V gives the comparison results. Meanwhile, it states the future research that can be continued.

CHAPTER II

LITERATURE REVIEW

2.1 Traditional Methods

As a common approach, Net Present Value (NPV) is ususally used to evaluate the investments or the real asset investments decisions. NPV is based on the Discounted Cash Flow (DCF) analysis. The DCF approach was proposed by Fisher (1907, 1930) when he pioneered the theory of interest and the value of time. It is a valuation method which can be used to estimate the attractiveness of an investment opportunity. After discounting the future cash flows back to the Present Value (PV), DCF analysis can help the managers to evaluate the potential value of the investment project. If the PV is higher than the current cost of the investment project, the managers may think that the investment opportunity is a good one. On the contrary, if the PV is less than the current cost, the managers may give up the investment opportunity.

After discounting the expected cash flows at a required rate of return, the NPV of an investment project can be calculated by the following formula

$$NPV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} - C_0, \tag{2.1.1}$$

where C_t is the net cash inflow during the period t, C_0 is the total initial investment cost, r is the discount rate, t is the number of time period, T is the total number of time periods or the life cycle of the investment project.

Considering the case of a series of investment expenditure, the NPV can be obtained from the difference between the PV of cash inflows and the PV of cash outflows,

$$NPV = \sum_{t=1}^{T} \frac{I_t}{(1+r_t)^t} - \sum_{t=0}^{T} \frac{O_t}{(1+R_t)^t}. \tag{2.1.2}$$

Here, I_t is the cash inflow at the period t, O_t is the cash outflow at the period t, r_t and R_t are the related discount rates, respectively.

Following these formula, the managers can make decisions. If $NPV > 0$, the investment may be economic and the investors may invest. If $NPV < 0$, the investment may be not feasible and the investors may stop.

Although the DCF method can be used widely, the traditional technique still has some limitations. The disadvantages have been increasingly recognized (Myers, 1984; Hodder et al., 1985; Trigeorgis et al., 1987; Brealey et al., 1992; Ross, 1995; Dixit and Pindyck, 1995). After analyzing the importance of a company's strategy in the capital budget process, Myers (1984) illustrated the limitations of DCF and recommended that the decision makers could get better decisions by options pricing rather than the DCF method. Hodder and Riggs (1985) thought that the DCF approach had been used incorrectly in practical applications. Since the project risk might decrease gradually with the project continuing and the management flexibility might also reduce the project risks, it should not use only one discount rate throughout the project's life cycle. Trigeorgis and Manson (1987) pointed out that when decision makers used traditional methods to make decisions, they usually assumed that the estimated future cash flows could be estimated on the premise of the future certainties. But in fact, the traditional approaches can not estimate the management flexibility in the investment decision-making process. Thus there will appear a biased result for an investment project by the NPV or DCF method in an uncertain environment.

The traditional methods are inappropriate for a rapidly changing investment environment, and they cannot reflect the available contingent decisions and the managerial flexibility to act on those decisions. The reason is that the traditional methods only consider two cases: investing immediately or giving up permanently. Dixit and Pindyck (1995) illustrated that if the managers might make an investment decision at a particular time without any change, they would ignore the value created by the delay of investment decisions, which might result in errors on the project value. In other words, the managers will make a decision-making error of the entire investment. In real life, the managers can usually wait until more information appears, then make the decisions to invest or not. More researches (Hayes and Abernthy, 1980; Hayes and Garvin, 1982; Trigerorgis and Mason, 1987; Trigerorgis, 1997; Tseng and Barz, 2002; Lewis et al., 2004) showed that the value of the future flexibility to expand, contract, or abandon could not be captured by DCF approach. Furthermore, Kodukula and Papudesu (2006) indicated that the NPV method was based on a set of fixed assumptions that related to the project payoff (a deterministic approach), whereas the payoff was uncertain and probabilistic.

2.2 Real Option Analysis

Real Option Analysis (ROA) offers new ways to fill the gaps that the traditional methods cannot address. The real option idea was originally developed from the financial option in the 1970s by Black and Scholes (1973) and Merton

(1973). However, the concept of real option was firstly proposed by Myers (1977). In his opinion, if a company can obtain a right after it has an investment decision, it can use the right to buy or sell the physical asset or the investment plan in the future. Meanwhile, Myers pointed out the similarities between the financial option and real option, but he thought that the project's value should be equal to the NPV of the project plus the value of the future option, when the investment project had a highly uncertain characteristic.

2.2.1 Concept of Real Option

Real Option (RO) is the extension of financial option theory to real assets. The main idea of ROA is to analyze the uncertainty of the projects, that is, it considers all the possible ranges of the cash flows. Based on the probability distribution of the cash flows and the expected information of the future market, the managers can make decisions with less subjective forecast of the future cash flow. Compared with the traditional methods, the results of real option analysis show that the uncertainties of investment opportunities accompany the greater investment value.

RO is the right - but not the obligation - to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. For example, the opportunity to expand a new production line can be considered as a real call option. The opportunity to reduce the scale of production can be seen as a real put option.

Although the idea of real option originates from the financial option, there still are some differences between them shown in Table 2.1. The first difference is that the financial options never have negative underlying asset values, but the real options can have. Secondly, the information about the valuation parameters in financial options are more easier available than that used in real options in the market. Another significant difference is the complexity of the real options relative to the financial options. In fact, real investments may have several interactive real options, whereas the financial options usually have straightforward payoff functions. Meanwhile, the length of the investment period is typically different, and the uncertainty changes more in the real option investment.

Table 2.1 Comparison of financial options and real options.

	Financial options	Real options
Underlying asset	The financial asset such as stock.	The real asset such as investment projects.
Current value	The current value of the financial asset such as the current price of a stock.	The present value of the real asset such as the present value of an investment project.
Option price	The price paid to acquire the option, which is fixed by the financial market.	The price paid to acquire or create the option, keep it alive and clear the uncertainty. The option price is not fixed, it can be usually negotiable.
Exercise price	The price paid to buy or sell a underlying financial asset, it is a fixed value defined in the option contract.	The cost of buying or selling the underlying real asset, or the amount of money to exercise the option.
Expiration time	It is defined in the option contract, and it is usually known clearly.	The time until the decision must be made. It is clearly known in some cases and not in others.
Volatility	It is the volatility or the standard deviation of the financial asset such as the volatility of a stock.	It is the uncertainty about the future value of the real asset, such as the probability distribution of the expected cash flow.
Discounted rate	The risk free interest rate.	The risk free discount rate.

2.2.2 Categories of Real Options

According to the differences of the investors' behaviors, Trigeorgis (1993) divided real options into seven basic categories, such as option to defer, option to abandon, option to switch, option to grow, option to interact, option to staged investment, and option to alter operating scale. However, with the development of real option theory, real option shows the trend of diversification. Here, based on the different investment choices, this subsection shows the concept of some categories in real options as follows.

Option to defer: Option to defer means that the option owners have the right - but not the obligation - to delay to make decisions by waiting for the new information about the market, such as the information about price, cost or other aspects.

For example, the managers have the option to make decisions such as launching the new product right now, or postponing it to the market in the future. If they launch the new product immediately, they will obtain cash flow earlier relative to waiting. On the contrary, if they delay, they may have time to find a better way to re-launch this product.

Option to abandon: Option to abandon means that the option owners have the right - but not the obligation - to give up the project, although it has already been implemented.

If the assets of a project are sold at the market, the market value of the project will be the value of this option. If the managers use these assets in other areas, the opportunity cost will be the value of this option. In general, it may be more preferable to continue to operate, or it may be better to terminate in some cases.

Option to switch: Option to switch means that the option owners have the right - but not the obligation - to convert between a variety of decisions in the future.

Due to the investment variability of a project, the corresponding switching option is usually contained in the initial design of the project. For example, the flexible production equipment allows the production line to switch easily between the different products. Once the managers decide to switch, the value of this option will depend on the states before and after the transformation.

Option to grow: Option to grow means that the option owners have the right - but not the obligation - to get some new investment opportunities, if the project has the initial success.

The growth option can constitute a value chain between related projects, so the implementation of the project will create broader space and more opportunities. Meanwhile, the project value does not depend on the size of the cash flow value generated by its own, but it is the performance of the future growth opportunities, such as providing a new generation of products, or the ability to carry out new investment projects in the future.

In practice, different investment projects may have different kinds of real options, and some investment projects may contain different types of real options at the same time.

2.3 Application of Real Option Analysis to Renewable Energy

Up to now, some scholars (Venetsanos et al., 2002; Davis and Owens, 2003; Yu et al., 2006; Kjaerland, 2007; Siddiqui et al., 2007; Bockman et al., 2008;

Kumbarolu et al., 2008; Munoz et al., 2009; Martinez and Mutale, 2011; Arenairo et al., 2011; Denis et al., 2014) had focused on the ROA to renewable energy investment. Most research consisted of new renewable power generation such as wind power, hydropower or solar photovoltaic power. These authors discussed the impact of uncertain factors on renewable energy investment. For example, the uncertain factors include non-renewable energy costs, renewable energy costs, research and development expenditure of renewable energy, abandonment and maintenance costs, the demand of renewable energy and so on. They also illustrated the decision-making process under different scenarios. Compared with the traditional methods, their results showed higher expected profits for projects planned with the advanced real options methodology.

Another important real option application to renewable energy is on the fuel ethanol projects. Using the ROA developed by Dixit and Pindyck (1994), Schmit et al. (2009) analyzed the investment and operating decisions of the corn-based dry-grind ethanol facilities. Kirby and Davison (2010) constructed a real option model to show the valuation of an ethanol plant as a spark spread between the corn price and gasoline price. Their analysis indicated that the value of an ethanol plant monotonically decreased with the correlation of the corn price and gasoline price increasing.

At present, some scholars (Lee and Shih, 2010, 2011; Schmit et al., 2011; Lin and Wessh, 2013; Zhang et al., 2014; Maxwell and Davison, 2014) have already constructed real option models to investigate the influence of the renewable energy policy. For example, Schmit, Luo and Conrad (2011) indicated that the effects of policy affecting ethanol plant revenues dominated the effects of those policies affecting production costs in USA. Regardless of the plant size, U.S. ethanol policy narrowed the distance between the optimal entry and exits curves. In absence of these policies, much of the recent expansionary periods would have not existed and the market conditions in the end of 1990s might have led to some plant closures. Zhang et al. (2014) used quadrinomial lattice tree to describe the non-renewable cost and carbon price. They used NPV to provide the functions of unit decision value at each node under government and investor perspective. Their work showed that the managerial flexibility and the unit decision value under ROA were underestimated by using NPV. However, the difference was very small, since the volatility rate of stochastic variables was relatively small and the speed of technological progress was not fast enough. Meanwhile, at the current level of subsidy, the government would suffer some losses. Maxwell and Davison (2014) quantified the impact of government policy of corn ethanol facilities and investigated the subsequent negative effects on firms. Based on the dynamic programming principle with Geometric Brownian Motion (GBM) stochastic differential equations, they showed the evidence of the increased correlation between corn and ethanol prices. The analytical solution of the partial differential equation simplified from a Bellman equation showed that dynamic programming can effectively assess the impact of the renewable energy policy on the revenue and cost.

In sum, these studies demonstrate deeply that the RO methods such as lattice tree and dynamic programming are suitable for evaluating the value of the renewable energy projects and investigating the influence of the related policies.

CHAPTER III

REAL OPTION MODEL

BASED ON LATTICE TREE METHOD

Real option analysis can be divided into two categories: numerical and analytic methods. Binomial lattice tree is a simple numerical method, which can be used to approximate the underlying stochastic process. In this Chapter, we will use binomial lattice tree approach to approximate the processes of two hypothetical stochastic variables, then construct a quadrinomial lattice tree in order to describe the option values. Specially, the quadrinomial lattice tree is called a bidimensional binomial lattice approach, which is named by Fan (2013).

3.1 Lattice Tree Method

The concept of lattice tree is based on the construction of a tree which starts from an initial value of the state variable. It includes the binomial tree, the ternary tree and other more complex cases. Cross et al. (1979) proposed the standard binomial option pricing model, which is known as the Cox-Ross-Rubinstein (CRR) binomial tree. In this approach, the underlying asset evolved from a risk-neutral binomial tree with constant logarithmic price spacing and constant volatility.

Assume that the asset price S_t follows a GBM process $dS_t = \mu S_t dt + \sigma S_t dB_t$, B_t is a Brownian Motion (BM), that is, in a probability space, the BM increments satisfy that $B_{t_1} - B_{t_0}$, $B_{t_2} - B_{t_1}$, … , $B_{t_m} - B_{t_{m-1}}$ are independent and each increment is the normally distribution with $E\left[B_{t_{j+1}} - B_{t_j}\right] = 0$ and $Var\left[B_{t_{j+1}} - B_{t_j}\right] = t_{j+1} - t_j$.

The CRR model usually requires three steps to find the option value.

Step 1: create the binomial tree of the asset price.

Given the starting value S_0 at time t_0 , then the set of possible prices at time t_j is

$$S_j^k = S_0 u^{j-k} d^k , \tag{3.1.1}$$

where $k = 0,1,\dots,j$ and $0 \le j \le N$. Here, k is the number of the price downward movements, $u = e^{\sigma\sqrt{\Delta t}}$ is the range of the price upward movements, $d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}}$ is the range of the price downward movements, $\Delta t = \frac{T}{N}$, and N is a positive integer. T is the expiration date of the option. The binomial lattice tree can be shown as Figure 3.1.

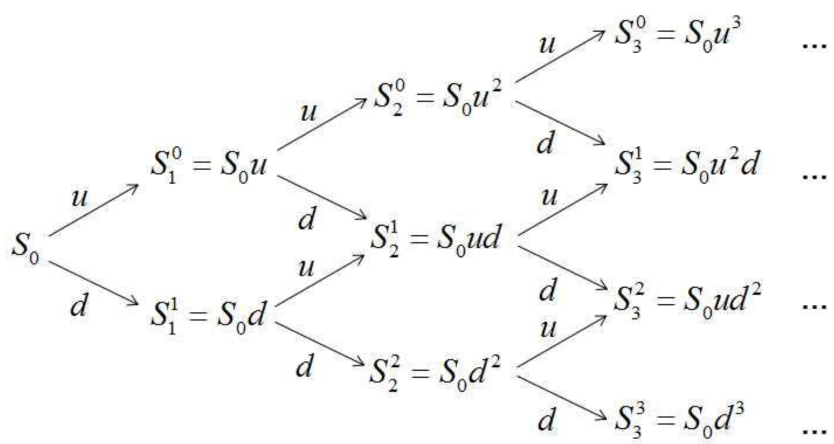


Figure 3.1 The diagram of the binomial lattice tree.

Step 2: find the option value at each final node.

At the expiration date $T = t_N$, the option price is usually known. With the European call and put option as examples, the option value of each final node is

$$\text{Call: } V_N^k = \max\left\{S_N^k - K, 0\right\}, \tag{3.1.2}$$

$$\text{Put: } V_N^k = \max\left\{K - S_N^k, 0\right\}. \tag{3.1.3}$$

Here, K is the strike price or exercise price.

Step 3: find the option value at the earlier node.

The option price at time t_j can be calculated by discounting the conditional expectation of the option price at the next time t_{j+1}, that is,

$$V_j^k = E\left[e^{-r\Delta t} V_{j+1} \middle| V_0, V_1, \ldots, V_j\right] = e^{-r\Delta t}\left[pV_{j+1}^k + (1-p)V_{j+1}^{k+1}\right], \tag{3.1.4}$$

where r is the risk-free interest rate, $p = \dfrac{e^{r\Delta t} - d}{u - d}$ is the risk-neutral probability of the asset price increase.

The CRR model is simple and easy to understand. With N increasing, the limit of the European CRR model is the famous Black-Scholes (B-S) formulas which can be shown as equation (3.1.5) to equation (3.1.9). The proof is shown in Appendix A.

$$\text{Call: } C = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2), \tag{3.1.5}$$

$$\text{Put: } P = K \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1), \tag{3.1.6}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)T\right], \tag{3.1.7}$$

$$d_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\frac{S}{K} + \left(r - \frac{\sigma^2}{2}\right)T\right], \tag{3.1.8}$$

$$N(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}dy \,. \tag{3.1.9}$$

Here, C is the European call option value, P is the European put option value, S is the current price of the financial asset, K is the strike price or excercise price, T is the expiration date, r is the risk-free interest rate, σ is the volatility of the financial option, $N(\cdot)$ is the cumulative standard normal distribution.

Boyle (1986) extended the binomial lattice tree to the trinomial lattice tree option pricing model, which is conceptually similar to the CRR model. The difference is that the trinomial lattice tree model has three jump parameters (up u, down d and stable or middle path m) and three related probabilities (p_u, p_d, p_m). In Boyle's trinomial lattice tree model, $u=e^{\lambda\sigma\sqrt{\Delta t}}$, $d=\frac{1}{u}=e^{-\lambda\sigma\sqrt{\Delta t}}$, $m=1$, where λ is constant. The basic idea of the trinomial lattice tree is as sketched in Figure 3.2. Meanwhile, the related probabilities can be calculated by these following formulas,

$$p_u=\frac{\left(e^{2r\Delta t}e^{\sigma^2\Delta t}-e^{r\Delta t}\right)e^{\lambda\sigma\sqrt{\Delta t}}-\left(e^{r\Delta t}-1\right)}{\left(e^{\lambda\sigma\sqrt{\Delta t}}-1\right)\left(e^{2\lambda\sigma\sqrt{\Delta t}}-1\right)}, \tag{3.1.10}$$

$$p_d=\frac{\left(e^{2r\Delta t}e^{\sigma^2\Delta t}-e^{r\Delta t}\right)e^{2\lambda\sigma\sqrt{\Delta t}}-\left(e^{r\Delta t}-1\right)e^{3\lambda\sigma\sqrt{\Delta t}}}{\left(e^{\lambda\sigma\sqrt{\Delta t}}-1\right)\left(e^{2\lambda\sigma\sqrt{\Delta t}}-1\right)}, \tag{3.1.11}$$

$$p_m=1-p_u-p_d \,. \tag{3.1.12}$$

Specially, the positive integer N must ensure that the probability $p_m \geq 0$. However, there will be great fluctuation in the convergence process. When $\lambda=\sqrt{2}$, the convergence will be better. Compared with the CRR model, the trinomial lattice tree is a better fit for the reality.

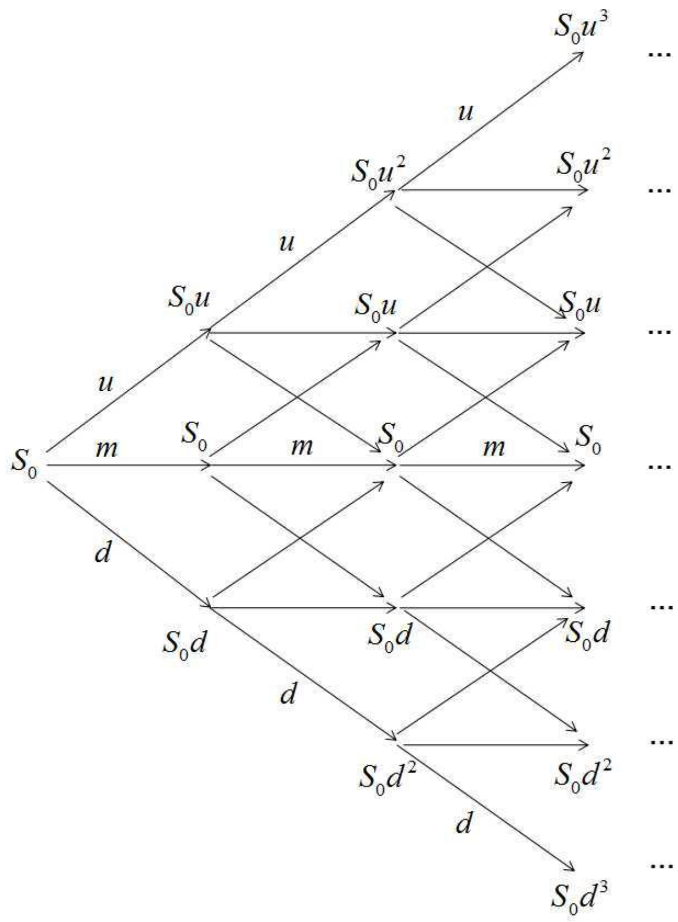


Figure 3.2 The diagram of the trinomial lattice tree.

Furthermore, Mandan et al. (1989) presented the N-tree model. The assumption of the N-tree model is about the underlying asset pricing options for some discrete stochastic process. So long as the tree structure of a price movement is formed, the value of the option can be extrapolated from the end node of the tree. The advantage of the N-tree model is that it can visually represent the underlying asset price movement.

3.2 Parameters and Parameters Estimation

According to the production process of the cellulosic ethanol project, the lattice tree model takes the main parameters such as the prices of the important raw materials and the products, and the subsidy level into consideration.

3.2.1 Parameters

There are two kinds of parameters: stochastic and non-stochastic parameters in the lattice tree model.

Category 1: stochastic parameters

The significant difference with other countries is that the cellulosic ethanol project and the price of fuel ethanol are controlled by the Chinese government. Based on the report of NDRC, the fuel ethanol price is set at 0.9111 times the price of No.93 gasoline from 1 May, 2011, so that the gasoline price is one of the key factors in the cellulosic ethanol investment project.

Suppose that the gasoline price follows GBM. Let $P^g(i,t)$ denote the price of gasoline with t periods elapsed and i downward movements, where $0 \le t \le T$, $0 \le i \le t$. T is the total number of time periods or the expiration date. In particular, the gasoline price of the next period can be presented as a binomial lattice tree as Figure 3.3 shows. Here, U_g is the range of the gasoline price upward movements. D_g is the range of the gasoline price downward movements. p_g is the risk-neutral probability of the gasoline increase.

$$
P^g(i,t) \begin{array}{l} \xrightarrow{p_g} P^g(i,t+1)=U_g\cdot P^g(i,t),\ 0\le t\le T, 0\le i\le t \\ \xrightarrow{1-p_g} P^g(i+1,t+1)=D_g\cdot P^g(i,t),\ 0\le t\le T, 0\le i\le t \end{array}
$$

Figure 3.3 The binomial lattice tree of the gasoline price.

As the main raw material, the price of corn cob is also assumed to follow GBM. Let $P^c(j,t)$ denote the corn cob price with t periods elapsed and j downward movements, where $0\le t\le T$, $0\le j\le t$. The binomial lattice tree of the corn cob price is shown in Figure 3.4. Similarly, U_c is the range of corn cob price upward movements. D_c is the range of the corn cob price downward movements. p_c is the risk-neutral probability of the corn cob increase.

$$
P^c(j,t) \begin{array}{l} \xrightarrow{p_c} P^c(j,t+1)=U_c\cdot P^c(j,t), 0\le t\le T, 0\le j\le t \\ \xrightarrow{1-p_c} P^c(j+1,t+1)=D_c\cdot P^c(j,t), 0\le t\le T, 0\le j\le t \end{array}
$$

Figure 3.4 The binomial lattice tree of the corn cob price.

Furthermore, these two stochastic variables are independent. Since the gasoline price is controlled by the Chinese government, the gasoline price is not affected significantly by the corn con price.

Category 2: non-stochastic parameters

Suppose that xylitol and pure lignin are the main by-products, zymin is another important raw material in the cellulosic ethanol project. Let P^x, P^l denote the prices of xylitol and pure lignin, P^z present the expense of zymin for every ton of cellulosic ethanol. Meanwhile, assume that the government must pay the cost of CO_2 emission, which is denoted by P^{cb}.

Since there are two construction stages in the cellulosic ethanol investment project by the basic assumption, let C_{other} denote the total construction costs (such as land, equipment and so on). In order to show the effects of the first stage construction cost, symbol a is used to represent the proportion of the stage-1 construction cost. Hence, the stage-1 construction cost J_1 and the stage-2 construction cost J_2 satisfy that

$$J_1 = aC_{other},$$

$$J_2 = (1-a)C_{other}.$$

Meanwhile, let symbol Q represent the capacity of the cellulosic ethanol project, and symbol S indicate the subsidy for every ton cellulosic ethanol from the Chinese government, which is stated by the government documents and the announcement of the first large-scale cellulosic ethanol producer. Furthermore, suppose that the cellulosic ethanol investment right will be lost if the construction program can not be completed on or before the expiration date T. The symbol r_f denotes the risk-free interest rate.

In order to show the impact of the main product and raw material to the decision value of the cellulosic ethanol investment, assume that all these parameters are considered to be constants.

3.2.2 Parameters Estimation

According to the Middle and Long Term Program of Renewable

Energy Development of China, the first large scale cellulosic ethanol project has been built in Shandong province. Thus the thesis uses the daily history data of No.93 gasoline price of Shandong province from 1 March, 2011 to 31 May, 2015. According to the logarithmic cash flow returns method (Kodukula and Papudesu, 2006), the volatility of the logarithm of gasoline price with the year as unit is $\sigma_g = 0.12$. Then the range of the gasoline price upward movements U_g can be calculated by the formula $U_g = e^{\sigma_g}$, so that $U_g = 1.12$. The range of the gasoline downward movements D_g satisfies $D_g = \frac{1}{U_g} = e^{-\sigma_g}$, thus $D_g = 0.89$. Furthermore, using the formula $p_g = \frac{e^{r_f} - D_g}{U_g - D_g}$, the risk-neutral probability of the gasoline price increase p_g equals 0.61. Using EXCEL, it is not hard to obtain the gasoline price at each node in its binomial lattice tree. Table 3.1 shows all these values in the binomial lattice tree of the gasoline price with initial data $P^g(0,0) = 8368$ yuan per ton. Here, $P^g(0,0)$ is the average price of the No. 93 gasoline in Shandong province from January to May in 2015.

Table 3.1 The value in the binomial lattice tree of the gasoline price (yuan/ton).

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$i=0$	8368	9372	10497	11756	13167	14747
$i=1$		7448	8341	9342	10463	11719
$i=2$			6628	7424	8315	9312
$i=3$				5899	6607	7400
$i=4$					5250	5880
$i=5$						4673

In Table 3.1, the value at node $(i,t+1)$ equals to $U_g = 1.12$ times the value at node (i,t), the value at node $(i+1,t+1)$ equals to $D_g = 0.89$ times the value at node (i,t).

Similarly, following the daily history data of the corn cob price from 1 March, 2012 to 31 May, 2015, the volatility of the corn cob price with the year as unit is $\sigma_c = 0.77$, the range of the corn cob price upward movements is $U_c = e^{\sigma_c} = 2.17$, the range of the corn cob price downward movements is $D_c = \frac{1}{U_c} = e^{-\sigma_c} = 0.46$, and the risk-neutral probability of the corn cob price increase is $p_c = \frac{e^{r_f} - D_c}{U_c - D_c} = 0.33$. Table 3.2 describes the values in the binomial lattice tree of the corn cob price with initial data $P^c(0,0) = 451$ yuan per ton. Here, $P^c(0,0)$ is the average price of the corn cob in Shandong province from January to May in 2015.

Table 3.2 The value in the binomial lattice tree of the corn cob price (yuan/ton).

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$j=0$	451	979	2124	4608	10000	21701
$j=1$		207	450	977	2120	4600
$j=2$			95	207	449	975
$j=3$				44	95	207
$j=4$					20	44
$j=5$						9

In this table, the value at node $(j,t+1)$ equals to $U_c = 2.17$ times the value at node (j,t), the value at node $(j+1,t+1)$ equals to $D_c = 0.46$ times the value at node (j,t).

The values of these non-stochastic parameters can be obtained easily from government documents, conference reports and company announcements. Assume that the price of xylitol is 23,000 yuan per ton, the price of pure lignin is 4,500 yuan per ton, producing one ton cellulosic ethanol needs 2,600 yuan zymin, the government pays 800 yuan subsidy to the investors and expense the carbon emission cost which average price is 50 yuan per ton.

By the announcements of the first large scale cellulosic ethanol producer LONGLIVE company in 2012, we suppose that the capacity 50,000 tons cellulosic ethanol project needs 166 million yuan as the total investment costs. Since there are two stages for the construction program and the proportion of the first construction stage cost a equals to 0.5, each stage cost is 83 million yuan, that is, $J_1 = J_2 = 83{,}000{,}000$ yuan.

Based on the treasury bond in China, the average interest of treasury bond in early 2015 is used to represent the risk-free interest rate. Hence, $r_f = 0.032$. From the Middle and Long Term Program of Renewable Energy Development of China, suppose that the cellulosic ethanol investment right will be lost if the construction program can not be completed on or before 2020, when is started from 2015. That is, the number of time periods (or expiration date) T is 5.

All these parameters and their estimated values are shown in Table 3.3.

Table 3.3 The parameters and estimated parameter values in the lattice tree model.

parameter	description	value	note
i	the number of gasoline price downward movements		
j	the number of corn cob price downward movements		
t	the number of time periods elapsed		
T	the total number of time periods	5	2015 to 2020
Q	the capacity of cellulosic ethanol	50,000 tons	some reports from NDRC of China
P^x	the price of xylitol	23,000 yuan/ton	The 6th stakeholder Plenary Meeting of EBTP
P^l	the price of pure lignin	4,500 yuan/ton	The 6th stakeholder Plenary Meeting of EBTP
P^z	the price of zymin	2,600 yuan/ton cellulosic ethanol	The 6th stakeholder Plenary Meeting of EBTP
P^{cb}	the average price of carbon	50 yuan/ton	the average price of carbon mitigation price from Jan. To May in 2015 based on China Beijing Environmental Exchange
S	the subsidy	800 yuan/ton cellulosic ethanol	some reports from NDRC of China
r_f	the risk-free interest rate	0.032	the average interest rate of treasury bonds in China in 2015 based on the Ministry of Finance the People's Republic of China
C_{other}	all the costs of the construction	166 millions yuan	
a	the proportion of the stage-1 construction cost	0.5	

Table 3.3 The parameters and estimated parameter values in the lattice tree model (Continued).

parameter	description	value	note
J_1	the cost of stage-1 construction	83 millions yuan	
J_2	the cost of stage-2 construction	83 millions yuan	
$P^g(0,0)$	the initial price of gasoline	8368 yuan/ton	the average price of No.93 gasoline price in Shandong province from Jan. to May. in 2015
$P^g(i,t)$	the price of gasoline at node (i,t)		
σ_g	the volatility of gasoline price	0.12	calculate with the history data between 1 March, 011 and 31 May, 2015
U_g	the range of gasoline price upward movements	1.12	$U_g = e^{\sigma_g}$
D_g	the range of gasoline price downward movements	0.89	$D_g = \frac{1}{U_g} = e^{-\sigma_g}$
p_g	the risk-neutral probability of the gasoline price increase	0.61	$p_g = \frac{e^{r_f} - D_g}{U_g - D_g}$
$P^c(0,0)$	the initial price of corn cob	451 yuan/ton	the average price of corn cob in Shandong province from Jan. To May. in 2015
$P^c(j,t)$	the price of corn cob at node (j,t)		
σ_c	the volatility of corn cob price	0.77	calculate with the history data between 1 March, 2012 and 31 May, 2015
U_c	the range of corn cob price upward movements	2.17	$U_c = e^{\sigma_c}$
D_c	the range of corn cob price downward movements	0.46	$D_c = \frac{1}{U_c} = e^{-\sigma_c}$
p_c	the risk-neutral probability of corn cob price increase	0.33	$p_c = \frac{e^{r_f} - D_c}{U_c - D_c}$

3.3 Quadrinomial Lattice Tree Model

Based on the binomial trees of stochastic variables P^g and P^c, we can establish a quadrinomial lattice tree model under government and investor perspectives.

3.3.1 Decision Tree

Since there are two construction stages before the project completion, the important assumptions are that each stage is irreversible and can be completed in one period.

Following the method of decision tree in Guthrie (2009), let label W stand for action "wait", label I stand for action "invest"; then the decision tree for the cellulosic ethanol investment with two construction stages can be shown in the following figure.

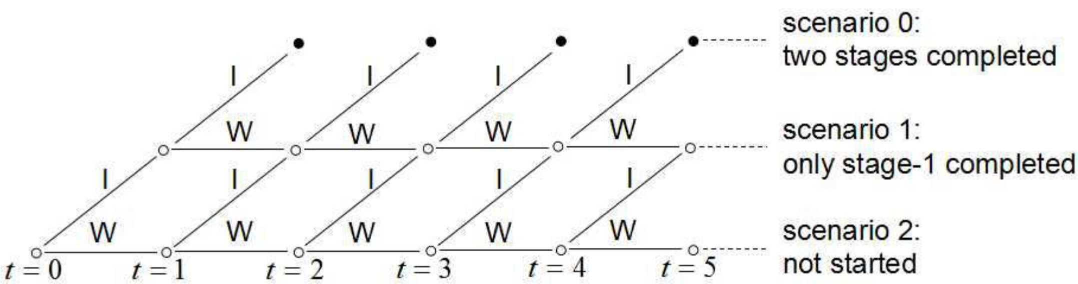


Figure 3.5 The decision tree.

As Figure 3.5 shows, the project owners face two choices at date 0 - wait or invest. If the owners wait, there will be zero cash flow at date 0 and none of the stages are started at date 1. If the owners invest in the stage-1 construction, there will be a cash flow $-J_1 = -aC_{other}$ at date 0 and only stage-2 construction remain at date 1. Thus, the stage-1 construction will be either completed or not at date 1. If the stage-1 construction has not been started, the owners face the same situation as at date 0. However, if the stage-1 construction has been completed, the owners must make the decision about the stage-2 construction, that is, choose action "wait" or action "invest" again. If they wait, there will be also zero cash flow at date 1 and only the stage-1 construction has been completed. On the other hand, if they undertake the stage-2 construction, there will be a capital expenditure $J_2 = (1-a)C_{other}$ at date 1 and these two stages will be completed at date 2. Starting from date 2, the project will be in one of three scenarios: two stages have been completed, only stage-1 has been completed and the project is not started, which are denoted as scenario 0, scenario 1 and scenario 2 separately. After expiration date T, the owners can do nothing since the investment right has expired.

3.3.2 Government and Investor Perspectives

According to Table 1.1 in Chapter I, producing one ton cellulosic ethanol needs $\frac{20}{3}$ tons corn cob. It also needs the purchase of the corresponding quantity of zymin P^z for every ton cellulosic ethanol in the production process. In addition, for one ton cellulosic ethanol, the project can also obtain $\frac{4}{5}$ tons xylitol and $\frac{2}{3}$ tons pure lignin at the same time. Moreover, the government pays the subsidy S to the investors. Specially, one ton fuel ethanol can be used instead of one ton gasoline in China, and one ton gasoline will release 3.15 tons CO_2 by the BP carbon emission calculator (the Chinese version is launched in early 2007 by BP company, which is one of the big oil and gas company in the world), so the government must offer the related emission cost $3.15P^{cb}$ for each ton cellulosic ethanol.

Following the similar idea that the decision value is the revenue minus the cost (Lee and Shih, 2010; Lee and Shih, 2011; Lin and Wessh, 2013; Zhang el at., 2014), the state variable values under government and investor perspectives can be shown as follows

Case 1: government perspective

$$X_G(i,j,t) = Q \cdot \left(0.9111P^g(i,t) + \frac{4}{5}P^x + \frac{2}{3}P^l - \frac{20}{3}P^c(j,t) - P^z - S - 3.15P^{cb} \right), \quad (3.3.1)$$

Case 2: investor perspective

$$X_I(i,j,t) = Q \cdot \left(0.9111P^g(i,t) + \frac{4}{5}P^x + \frac{2}{3}P^l - \frac{20}{3}P^c(j,t) - P^z + S \right), \quad (3.3.2)$$

where $0 \le t \le T$, $0 \le i, j \le t$. $X_G(i,j,t)$ is the market value of the completed cellulosic ethanol project at decision node (i,j,t) under government perspectives, $X_I(i,j,t)$ is the market value of the completed cellulosic ethanol project decision node (i,j,t) under investor perspectives, and other symbols are defined in Table 3.3.

3.3.3 Scenario Functions

Since there are two state variables, that is, variable i represents the gasoline price and variable j represents the corn cob price, the lattice tree real option model will incorporate multiple state variables. To do this, the research generalizes the notion of Guthrie (2009). Based on the decision tree, let $V_n(i,j,t)$ denote the market value of the investment right at date t if there are i downward movements in the first state variable and j downward movements in the second one. Here, $n=0,1,2$ represents the number of the construction stage remaining to be completed. That is, $V_0(i,j,t)$ presents the decision value with two stages completed, $V_1(i,j,t)$ describes the decision value with only the stage-1 completed, $V_2(i,j,t)$ denotes the decision value with the investment not started. Here, these symbols ignore the subscript G and I standing for the government and investor perspectives.

Scenario 0: two stages completed

If the construction program can be completed immediately, the investment right value is the value of the completed project. Therefore, the decision value functions under government and investor perspectives can be written as

$$V_{G0}(i,j,t) = -aC_{other} - e^{-r_f}(1-a)C_{other} + X_G(i,j,t), \tag{3.3.3}$$

$$V_{I0}(i,j,t) = -aC_{other} - e^{-r_f}(1-a)C_{other} + X_I(i,j,t), \tag{3.3.4}$$

where $0 \le t \le T$, $0 \le i, j \le t$. Here, a is the proportion of the stage-1 construction cost, C_{other} is the total construction cost, r_f is the risk-free interest rate.

Scenario 1: only stage-1 completed

Since the investment right will be lost if the construction program can not be completed on or before the expiration date T. The decision value must satisfy the following terminal conditions as

$$V_{G1}(i,j,T) = 0, \tag{3.3.5}$$

$$V_{I1}(i,j,T) = 0, \tag{3.3.6}$$

where $0 \le i, j \le T$.

For each date, the gasoline price may increase with probability p_g or decrease with probability $1-p_g$. Meanwhile, the corn cob price may increase with probability p_c or decrease with probability $1-p_c$. Hence, there are four cases at the next date as Figure 3.6 shown, which ignores the subscripts G and I.

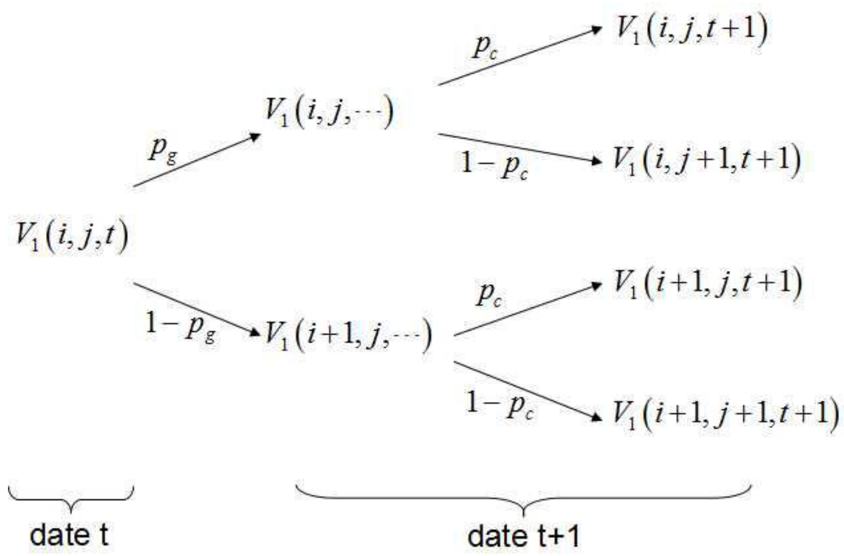


Figure 3.6 The stochastic decision-making process with two stochastic variables.

By backward induction, the lattice trees for the decision value V_1 under government and investor perspectives can be filled by

$$\begin{aligned} V_{G1}(i,j,t) = \max\{&-J_2 + X_G(i,j,t), \\ &e^{-r_f}\left[p_g\left[p_c V_{G1}(i,j,t+1) + (1-p_c)V_{G1}(i,j+1,t+1)\right]\right. \\ &\left.+(1-p_g)\left[p_c V_{G1}(i+1,j,t+1) + (1-p_c)V_{G1}(i+1,j+1,t+1)\right]\right]\}, \end{aligned} \tag{3.3.7}$$

$$\begin{aligned} V_{I1}(i,j,t) = \max\{&-J_2 + X_I(i,j,t), \\ &e^{-r_f}\left[p_g\left[p_c V_{I1}(i,j,t+1) + (1-p_c)V_{I1}(i,j+1,t+1)\right]\right. \\ &\left.+(1-p_g)\left[p_c V_{I1}(i+1,j,t+1) + (1-p_c)V_{I1}(i+1,j+1,t+1)\right]\right]\}, \end{aligned} \tag{3.3.8}$$

where $0 \le t \le T$, $0 \le i,j \le t$.

Thus, the government and investors can choose the action which can yield the maximum market value of the project.

Scenario 2: not started

Similarly, the decision value at scenario 2 has the same terminal conditions. Since the construction program has never been started, the project right value must equal 0 for both government and investors at expiration date T . For all i, j satisfying $0 \le i, j \le T$, then

$$V_{G2}(i, j, T) = 0, \tag{3.3.9}$$

$$V_{I2}(i, j, T) = 0. \tag{3.3.10}$$

The last line of lattice trees for V_2 can be filled by the terminal conditions. Furthermore, the decision values of the investment right at each earlier node can be calculated by backward induction based on the following equations.

$$\begin{aligned} V_{G2}(i, j, t) = \max\Big\{ & -J_1 + e^{-r_f}\Big[p_g\left[p_c V_{G1}(i, j, t+1) + (1-p_c)V_{G1}(i, j+1, t+1)\right] + \\ & (1-p_g)\left[p_c V_{G1}(i+1, j, t+1) + (1-p_c)V_{G1}(i+1, j+1, t+1)\right]\Big], \\ & e^{-r_f}\Big[p_g\left[p_c V_{G2}(i, j, t+1) + (1-p_c)V_{G2}(i, j+1, t+1)\right] + \\ & (1-p_g)\left[p_c V_{G2}(i+1, j, t+1) + (1-p_c)V_{G2}(i+1, j+1, t+1)\right]\Big]\Big\}, \end{aligned} \tag{3.3.11}$$

$$\begin{aligned} V_{I2}(i, j, t) = \max\Big\{ & -J_1 + e^{-r_f}\Big[p_g\left[p_c V_{I1}(i, j, t+1) + (1-p_c)V_{I1}(i, j+1, t+1)\right] + \\ & (1-p_g)\left[p_c V_{I1}(i+1, j, t+1) + (1-p_c)V_{I1}(i+1, j+1, t+1)\right]\Big], \\ & e^{-r_f}\Big[p_g\left[p_c V_{I2}(i, j, t+1) + (1-p_c)V_{I2}(i, j+1, t+1)\right] + \\ & (1-p_g)\left[p_c V_{I2}(i+1, j, t+1) + (1-p_c)V_{I2}(i+1, j+1, t+1)\right]\Big]\Big\}, \end{aligned} \tag{3.3.12}$$

where $0 \le t \le T$, $0 \le i, j \le t$.

All these equations constitute the real option model based on the quadrinomial lattice tree method.

3.4 Basic Analysis

3.4.1 Scenarios Analysis

The purpose of the scenario analysis is to analyze the benefit of the renewable energy subsidy policy to the cellulosic ethanol plants under government and investor perspectives. If these two construction stages have been completed, the decision values of the the project are shown as Table 3.4. If only the stage-1 construction has been completed, the decision values of the project are presented as Table 3.5. Meanwhile, if the project is not started, the decision values of the project are shown as Table 3.6.

Case 1: government perspective

Table 3.4 indicates that the initial decision value equals 959 million yuan in 2015 when all two stages are completed. The decision values from 2015 to 2017 are greater than zero, which reflects that the government will obtain the benefit during these periods, although the government pays the carbon emission cost and the subsidy. However, with the time elapsing from 2018 to 2020, the decision values are below zero if the gasoline price decreases 0 times. It is clear that if the cost of raw materials is too high, the benefit is much lower. If the government completes the stage-1 construction, the decision value increases from 959 to 1127 million yuan in 2015 (see Table 3.4 and Table 3.5). Thus the government can make an optimal decision after observing the movements of the gasoline price and the corn cob price when the stage-1 construction can be completed one year in advance. Based on scenario 2, if the government does not start the project, the decision value decreases from 1127 to 1028 million yuan (see Table 3.5 and Table 3.6), which is also more than 959 million yuan under scenario 0 (see Table 3.4). Table 3.6 indicates that it is better to choose action “invest” earlier under government perspective. Objectively, it is not optimal to invest in the project during the last two years.

In the following tables, the value at the year 2015+ k ($k = 1, 2, 3, 4, 5$) means that both gasoline price and corn cob price can decrease k times. From top to bottom, the first $k+1$ rows stand for the gasoline price decreases 0 times, the second $k+1$ rows stand for the gasoline price decreases 1 time, and so on. In each $k+1$ rows, the values from the first line to the $k+1$ line present the corn cob price decreases from 0 to k times.

Table 3.4 The decision values of the cellulosic ethanol investment at scenario 0 (million yuan, G = government, I = investors).

2015		2016		2017		2018		2019		2020	
G	I	G	I	G	I	G	I	G	I	G	I
959	1047	829	917	499	586	-271	-183	-2004	-1917	-5833	-5745
		741	829	400	488	-381	-293	-2128	-2040	-5971	-5883
		1086	1174	322	410	-469	-381	-2225	-2138	-6080	-5992
		998	1086	1056	1144	-538	-450	-2303	-2215	-6167	-6079
				958	1046	938	1026	-2365	-2277	-6236	-6149
				880	968	828	916	621	709	-6291	-6204
				1175	1262	741	829	498	586	-132	-44
				1076	1164	671	759	400	488	-270	-182
				998	1086	1195	1283	323	410	-380	-292
						1085	1173	261	349	-467	-379
						997	1085	1178	1266	-536	-448
						928	747	1055	1143	-591	-503
						1249	1337	957	1045	1075	1163
						1139	1227	879	967	937	1025
						1052	1140	818	905	827	915
						982	1070	1296	1384	740	828
								1173	1261	671	759
								1075	1163	616	704
								997	1085	1331	1419
								936	1024	1193	1281
								1321	1409	1084	1171
								1198	1286	996	1084
								1100	1188	927	1015
								1022	1110	872	960
								961	1049	1385	1473
										1247	1335
										1138	1226
										1051	1139
										982	1069
										926	1014
										1397	1485
										1259	1347
										1149	1237
										1062	1150
										993	1081
										938	1026

Table 3.5 The decision values of the cellulosic ethanol investment at scenario 1 (million yuan, G = government, I = investors).

2015		2016		2017		2018		2019		2020	
G	I	G	I	G	I	G	I	G	I	G	I
1127	1127	1040	1026	863	835	517	481	0	0	0	0
		938	941	757	747	427	408	0	0	0	0
		1261	1254	673	678	356	349	0	0	0	0
		1154	1167	1245	1225	300	303	0	0	0	0
				1126	1126	1145	1106	0	0	0	0
				1031	1048	1011	996	855	790	0	0
				1359	1343	905	909	705	667	0	0
				1240	1245	821	840	586	569	0	0
				1145	1167	1393	1363	492	491	0	0
						1259	1253	416	429	0	0
						1153	1166	1393	1347	0	0
						1069	1096	1243	1223	0	0
						1446	1417	1124	1125	0	0
						1312	1307	1030	1048	0	0
						1206	1220	955	986	0	0
						1121	1151	1507	1465	0	0
								1358	1341	0	0
								1239	1244	0	0
								1144	1166	0	0
								1069	1104	0	0
								1532	1490	0	0
								1382	1366	0	0
								1263	1269	0	0
								1168	1191	0	0
								1093	1129	0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0

Table 3.6 The decision values of the cellulosic ethanol investment at scenario 2 (million yuan, G = government, I = investors).

2015		2016		2017		2018		2019		2020	
G	I	G	I	G	I	G	I	G	I	G	I
1028	1026	957	943	780	752	434	398	0	0	0	0
		855	858	674	664	344	325	0	0	0	0
		1152	1143	590	595	273	266	0	0	0	0
		1046	1055	1136	1114	217	220	0	0	0	0
				1017	1016	1038	997	0	0	0	0
				923	938	904	887	0	0	0	0
				1250	1231	798	800	0	0	0	0
				1131	1133	714	730	0	0	0	0
				1036	1055	1284	1251	0	0	0	0
						1150	1141	0	0	0	0
						1044	1054	0	0	0	0
						960	985	0	0	0	0
						1336	1305	0	0	0	0
						1203	1195	0	0	0	0
						1096	1108	0	0	0	0
						1012	1039	0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
								0	0	0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0
										0	0

Case 2: investor perspective

Comparing these three tables, under investor perspective, the initial decision value at scenario 0 equals 1047 million yuan (see Table 3.4) , which is less than 1127 million yuan at scenario 1 (see Table 3.5), but it is a little more than 1026 million yuan at scenario 2 (see Table 3.6) in 2015. Table 3.4 indicates the same phenomenon under the government perspective. If the investors have completed the construction immediately, the decision values from 2015 to 2017 are greater than zero as well. With the time elapsing from 2018 to 2020, the decision values are less than zero when the gasoline price decreases 0 time. Clearly, the benefit decreases with the costs increasing. Table 3.5 shows that it is better to complete the stage-1 construction one year in advance as well. At scenario 2, although the government offers a subsidy for the cellulosic ethanol project, the investors are still better off to choose action "wait" during the last two years.

In addition, the managerial flexibility can be well underestimated using real option analysis under government and investor perspectives. The shaded areas in Table 3.4 present that the government and investors will suffer losses if all the construction stages have been completed at these nodes. The shaded areas in the Table 3.5 and Table 3.6 indicate that choosing action "wait" is better at these nodes.

3.4.2 Effects of the Subsidy

Set the subsidy for cellulosic ethanol to different numerical values (0,200,400,600,800,1000,1200,1400,1600,1800), and the unit of the decision value in the following figures is million yuan. Figure 3.7 to Figure 3.9 show the changes at each scenario under government and investor perspectives.

Figure 3.7 shows the change of the initial decision value in 2015 under government and investor perspectives at scenario 0. With the subsidy increasing from 0 to 1800, the decision value decreases for government but increases for investors, if all the construction stages have been completed.

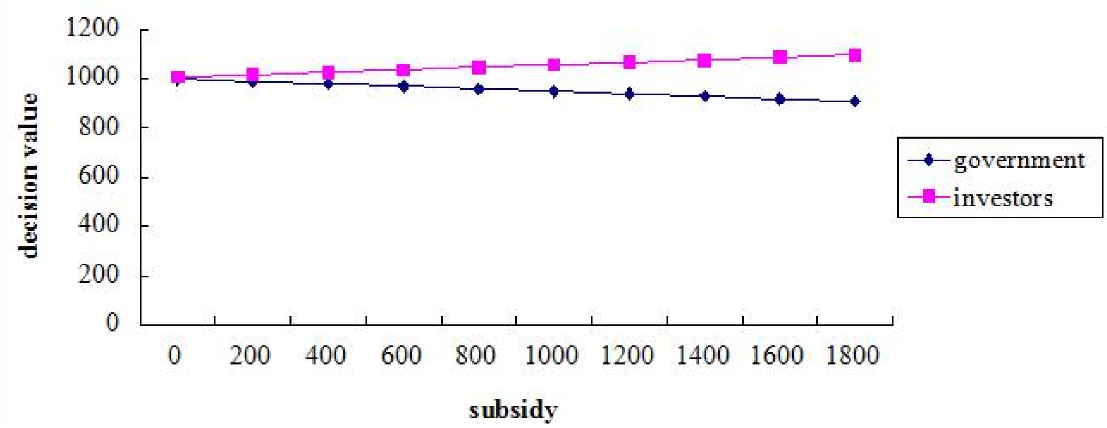


Figure 3.7 The change of the initial decision value at scenario 0 with different subsidy.

Figure 3.8 and Figure 3.9 indicate the changes of the initial decision value in 2015 at the scenario 1 and scenario 2 under government and investor perspectives. These two figures show the same regularity as scenario 0. The decision value decreases for government but increases for investors with the subsidy increasing. Furthermore, whether the stage-1 construction is completed or not, the decision value curves intersect at the subsidy 800 yuan (see Figure 3.8 and Figure 3.9).

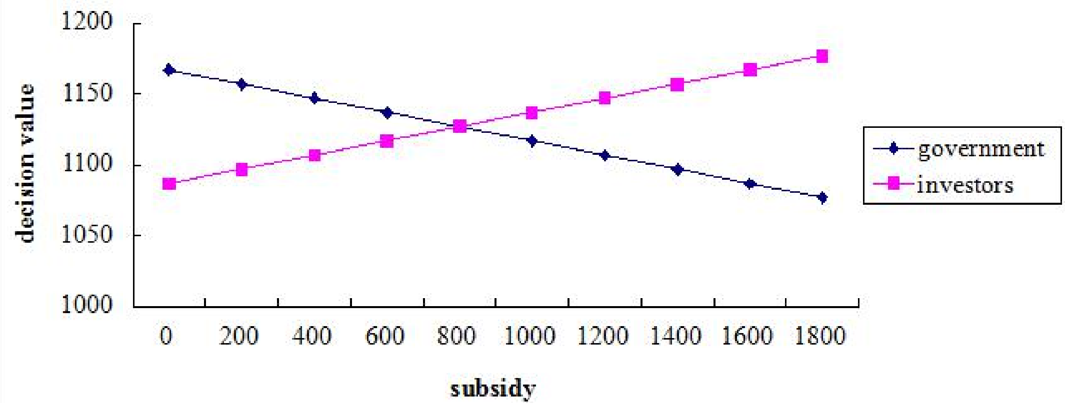


Figure 3.8 The change of the initial decision value at scenario 1 with different subsidy.

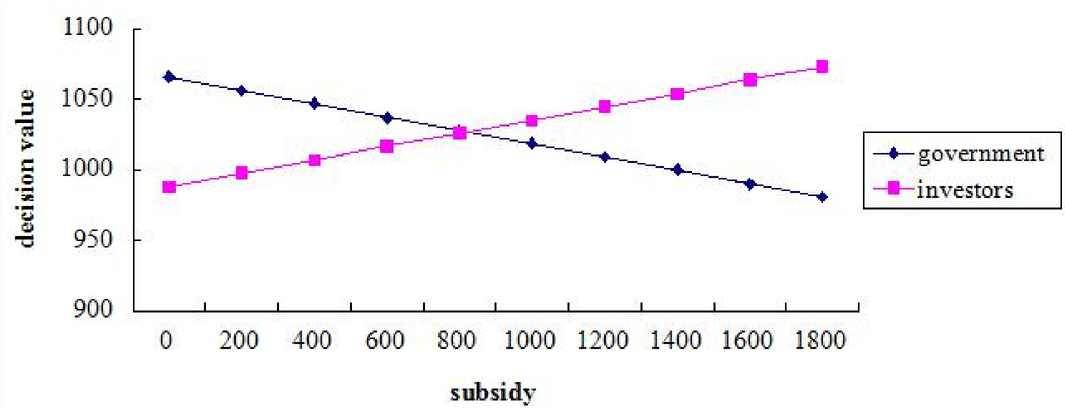


Figure 3.9 The change of the initial decision value at scenario 2 with different subsidy.

If people exploit the high value by-products such as xylitol and pure lignin in cellulosic ethanol project, reducing the subsidy will not affect the decision values too much. However, if there does not exist any by-product, although the initial decision values at scenario 1 and scenario 2 are positive, the government and the investors will not invest immediately, since the initial decision values are negative in 2015 if the project has already been completed. In this case, the subsidy plays a certain promotion role to the investors, because it can increase the benefit to investors. Of course, it increases the loss of the government to a certain extent. These decision values at the beginning year 2015 are shown in Table 3.7.

Table 3.7 The initial decision values with no by-product (million yuan).

	$s=800 \quad a=0.5$		$s=0 \quad a=0.5$	
	government	investors	government	investors
scenario 0	-110	-22	-70	-62
scenario 1	158	151	187	122
scenario 2	91	84	119	60

Hence, reducing subsidy will ease the loss of government and cut down the benefit of investors, but enhancing subsidy is good for the promotion of renewable energy investment at the beginning stage. The result seems similarly in the reality.

3.4.3 Effects of the Proportion of the Stage-1 Construction Cost

According to the equations (3.3.3) and (3.3.4) at scenario 0, (3.3.7) and (3.3.8) at scenario 1, (3.3.11) and (3.3.12) at scenario 2, the proportion a of the stage-1 construction cost shows that the capital expenditure profile will affect the decision value. Figure 3.10 and Figure 3.11 present the changes of the initial decision value under government and investor perspectives with subsidy $s=800$ and by-products. These two pictures show a similar phenomenon. For both government and investors in 2015, with the proportion of the stage-1 construction cost increasing, the decision value decreases at scenario 0 , increases at scenario 1 and decreases at scenario 2. But the variation is more significant to the investors than to the government. Thus, the capital expenditure profile gives more risk to the investors than to the government.

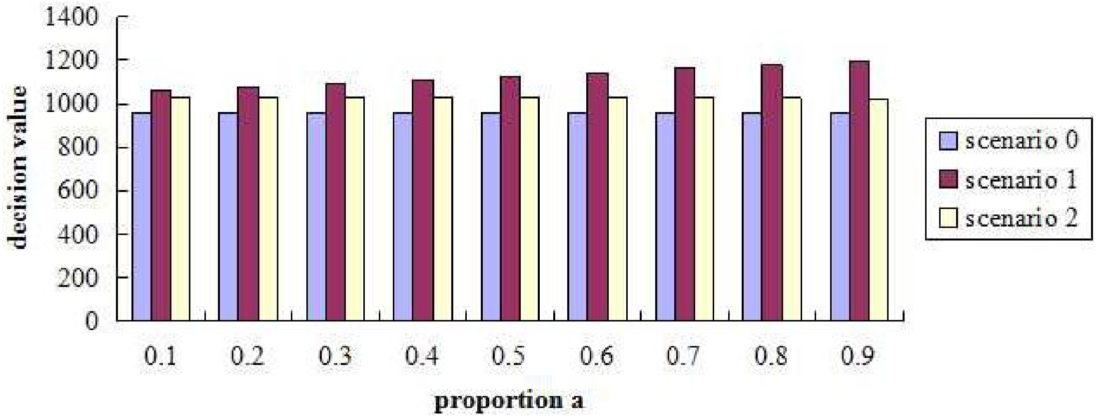


Figure 3.10 The change of the initial decision value under government perspective with s=800 and by-products.

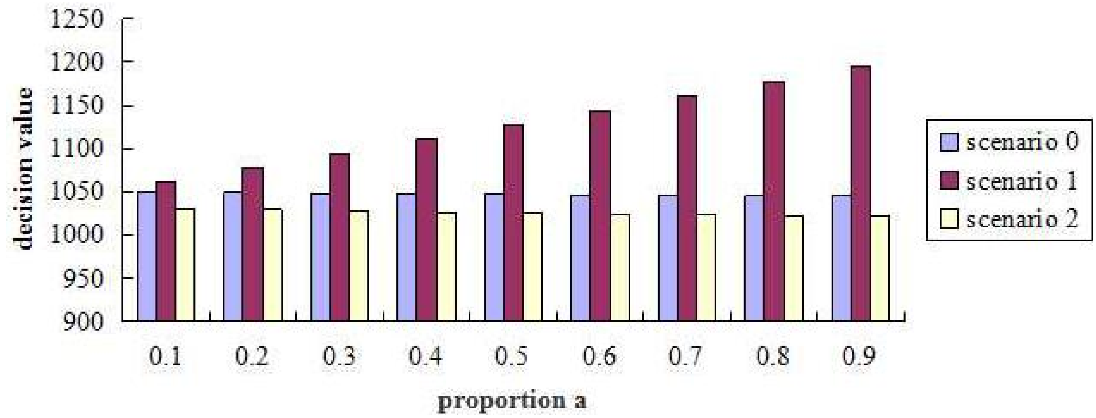


Figure 3.11 The change of the initial decision value under investor perspective with s=800 and by-products.

If there do not exist any high value by-products in the cellulosic ethanol project, the variation of the proportion a shows more influence to the decision for both government and investors, which can be shown in Figure 3.12 and Figure 3.13. In this case, under both perspectives, the decision values are negative no matter how the proportion a changes at scenario 0. Meanwhile, the decision value increases significantly at scenario 1, but only decreases a little at scenario 2. Thus, the capital expenditure profile indicates more risk to both government and investors.

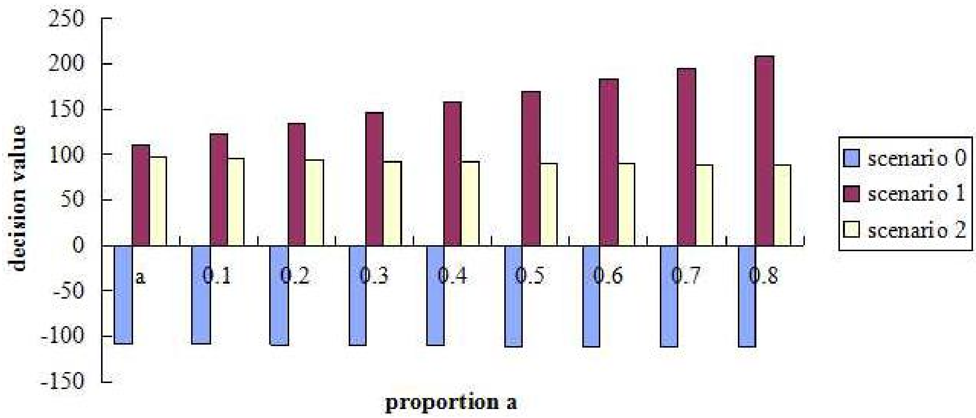


Figure 3.12 The change of the initial decision value under government perspective with s=800 and no by-product.

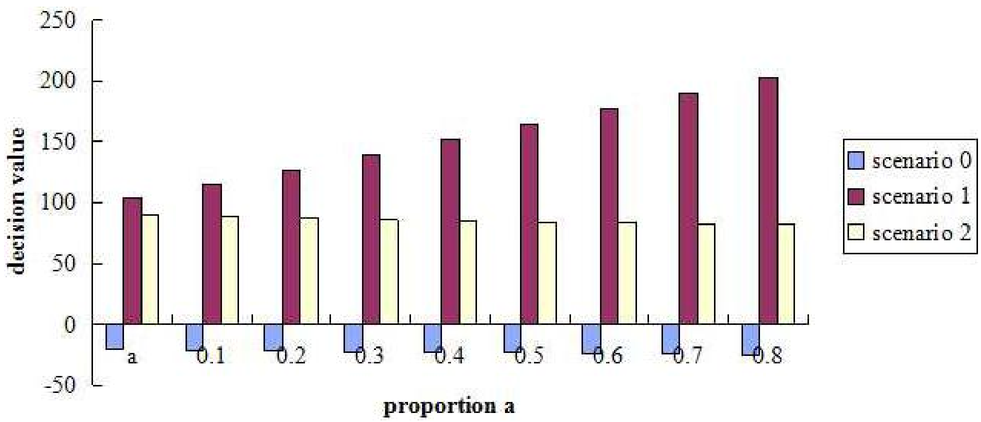


Figure 3.13 The change of the initial decision value under investor perspective with s=800 and no by-product.

Based on the Figure 3.10 to Figure 3.13, relative to the subsidy, the existence of by-product shows more obvious influence on the decision value. Meanwhile, no matter how the proportion a changes, improving the technology of cellulosic ethanol, making full use of the raw materials and finding high value by-product are effective ways to enhance the benefits.

CHAPTER IV

REAL OPTION MODEL

BASED ON DYNAMIC PROGRAMMING METHOD

Dynamic programming is a general and useful tool for the dynamic optimization problems under uncertainty conditions in real option analysis. It can be used to derive the analytic solution for a real option model. The dynamic programming method will be applied to derive the option value functions in the infinite time horizon case in this chapter.

4.1 Dynamic Programming Method

The dynamic programming method is based on splitting the decisions into parts that comprise a sequence in time, and it aims to find the optimal path of decisions. It usually breaks a whole sequence of decisions into two components: the immediate decision and some future decisions. Since there is no decision pending at the last decision point, working backwards can derive the optimal path starting from the initial decision point.

The time in dynamic programming method can be considered as either discrete or continuous. The binomial lattice tree is a typical discrete time case of the dynamic programming, which has been shown in Chapter III.

Let x_t be the state variable at time period t, and x_t be a Markov process.

A Markov process is a stochastic process that satisfies the Markov property, that is, the conditional probability distribution of the processes future states (conditional on both past and present states) depends only upon the present state, not on the sequence of events that preceded it. It can be thought of as "memoryless".

At each period t, the asset owners are able to make some choices for the operation of the asset. Let $\pi_t(x_t, u_t)$ and $F_t(x_t)$ denote the immediate profit flow and the value of the asset, respectively. Here, u_t represents the control variable of these choices. Furthermore, assume that r is a constant discount rate, then the asset value satisfies the following Bellman equation (Dixit and Pindyck, 1994) at each time period t,

$$F_t(x_t) = \max_{u_t} \left\{ \pi_t(x_t, u_t) + \frac{1}{1+r} E_t \left[F_{t+1} [x_{t+1}] \right] \right\},$$

where $E_t[\cdots]$ is the expectation operator at time t based on a real world measure. The Bellman equation is also called a dynamic programming equation or an optimality equation, which is named by its developer - an American applied mathematician Richard E. Bellman (August 26, 1920 - March 19, 1984). Clearly, this Bellman equation means that the investors can choose the optimal action that can bring the maximum value with one special control variable.

Furthermore, if the profit and the asset value are defined as functions of the state variable x, the optimization is the maximization of a Bellman equation that is satisfied by the asset value with constant discount rate. Assume that the state variable x follows a stochastic process defined as a Stochastic Differential Equation (SDE)

$$dx = a(x,t)\,dt + b(x,t)\,dB_t,$$

where B_t is a BM. If the managers can either continue waiting to receive a cash flow $\pi(x,t)$, or exercise the investment to get the payoff $\Omega(x,t)$, then the Bellman equation can be presented as

$$F(x,t)=\max\left\{\Omega(x,t),\pi(x,t)\Delta t+\frac{1}{1+r\Delta t}E_t\left[F\left[x+\Delta x,t+\Delta t\right]\right]\right\}.$$

The first part stands for the value when the investors take one action such as selling the project. The second part shows the value when the investors choose another action such as continuing to produce.

Considering the part $F(x,t)=\pi(x,t)\Delta t+\frac{1}{1+r\Delta t}E_t\left[F\left[x+\Delta x,t+\Delta t\right]\right]$; since $F(x,t)$ is known to be related to the future time $t+\Delta t$, then

$$rF(x,t)\Delta t=\pi(x,t)\Delta t+r\pi(x,t)(\Delta t)^2+E_t\left[F\left[x+\Delta x,t+\Delta t\right]-F(x,t)\right].$$

By the limitation $\Delta t\to dt$, $F\left[x+\Delta x,t+\Delta t\right]-F(x,t)\to dF(x,t)$, then

$$rF(x,t)dt=\pi(x,t)dt+E_t\left[dF\right].$$

Applying Ito's Lemma, which is introduced in Appendix B,

$$dF(x,t)dt=F_x(x,t)dx+F_t(x,t)dt+\frac{1}{2}F_{xx}(x,t)(dx)^2,$$

where $(dB_t)^2=dt$, $dtdB_t=0$, $(dt)^2=0$, $(dx)^2=b^2(x,t)dt$. Hence, the Bellman equation can be modified and simplified as a Partial Differential Equation (PDE) as follows

$$\frac{1}{2}b^2(x,t)F_{xx}(x,t)+a(x,t)F_x(x,t)+F_t(x,t)-rF(x,t)+\pi(x,t)=0 .$$

Here, the subscripts represent the related partial derivatives. Specially, this PDE needs to satisfy two classic boundary conditions, the value-matching condition and the smooth-pasting boundary condition,

$$F\left(x^*(t),t\right)=\Omega\left(x^*(t),t\right),$$

$$F_x\left(x^*(t),t\right)=\Omega_x\left(x^*(t),t\right),$$

where $x^*(t)$ is the threshold value at which point the investment is triggered. These two boundary conditions come from the economic considerations. This implies that the values of $F(x,t)$ and $\Omega(x,t)$ will meet tangentially at the boundary $x^*(t)$ for the reason of maintaining continuity. With these two boundary conditions, the threshold value $x^*(t)$ and the value function $F(x,t)$ can be jointly solved.

The dynamic programming method is easier in incorporating operational constraints, but the usage of a subjective discount rate may lead to a valuation result which deviates from the real market price of the asset. Furthermore, it is not very easy to obtain the threshold value and the value function.

4.2 Parameters and Parameters Estimation

4.2.1 Parameters

Most of the parameters are the same as used in Section 3.2.1 in Chapter III, but there still are some differences. The first difference is that the symbol r denotes the discount rate. Although the discount rate is usually different from the risk-free interest rate, we still use identical values, which can be calculated by the average interest of treasury bond in early 2015 in China. The second difference is about the stochastic variables. In the dynamic programming real option model, the unit revenue P is considered as a whole which includes the stochastic variable P^g

and the other related non-stochastic variables such as P^x and P^l. Likewise, the unit expenditure C is seemed as a whole as well, it contains the stochastic variable P^c and some other related non-stochastic variables such as P^z.

Thus, all these parameters used in Chapter IV can be found in Table 3.3 in Chapter III.

4.2.2 Parameters Estimation

In the infinite time horizon case, assume that the unit revenue P and the unit expenditure C (which includes all these expenses but excepts the investment construction cost) are stochastic variables, and these two stochastic variables follow GBM processes,

$$dP = \mu_P P dt + \sigma_P P dB_{1t}, \tag{4.2.1}$$

$$dC = \mu_C C dt + \sigma_C C dB_{2t}, \tag{4.2.2}$$

$$dB_{1t} dB_{2t} = \rho dt. \tag{4.2.3}$$

Here, μ_P is the drift of the unit revenue P, σ_P is the volatility of the unit revenue P, μ_C is the drift of the unit expenditure C, σ_C is the volatility of the unit expenditure C, B_{1t} and B_{2t} are BMs, ρ is the correlation coefficient of B_{1t} and B_{2t}.

By assumptions (4.2.1) and (4.2.2), the logarithms of P and C satisfy the following stochastic processes

$$d\ln P = \left(\mu_P - \frac{1}{2}\sigma_P^2\right)dt + \sigma_P dB_{1t}, \tag{4.2.4}$$

$$d\ln C = \left(\mu_C - \frac{1}{2}\sigma_C^2\right)dt + \sigma_C dB_{2t}. \tag{4.2.5}$$

Discretizing the stochastic process (4.2.4) can be considered as

$$\Delta p_j = p_{j+\Delta t} - p_j = \left(\mu_P - \frac{1}{2}\sigma_P^2 \right)\Delta t + \varepsilon_{j+\Delta t} , \tag{4.2.6}$$

where the error $\varepsilon_{j+\Delta t} \sim N\left(0, \sigma_P^2 \Delta t\right)$. Here, p_j is the logarithm of P_j, that is, $p_j = \ln P_j$. Clearly, $E\left[\Delta p_j\right] = \left(\mu_P - \frac{1}{2}\sigma_P^2 \right)\Delta t$, $Var\left[\Delta p_j\right] = \sigma_P^2 \Delta t$.

Following the method introduced in Guthrie (2009, p266), Section 12.1.1, these parameters μ_P, σ_P can be estimated by the process

$$\Delta p_j = p_{j+1} - p_j = v_1 + \varepsilon_{j+1} ,$$

where the error $\varepsilon_{j+1} \sim N\left(0, \phi_1^2\right)$, v_1 is the sample mean of the array Δp_j, and ϕ_1 is the sample standard deviation of the array Δp_j. Then the estimated values of $\hat{v}_1$ and $\hat{\phi}_1$ can be obtained as follows

$$\hat{v}_1 = \left(\mu_P - \frac{1}{2}\sigma_P^2 \right)\Delta t ,$$

$$\hat{\phi}_1^2 = \sigma_P^2 \Delta t .$$

Hence, the estimated values of μ_P, σ_P are

$$\hat{\mu}_P = \frac{\hat{v}_1}{\Delta t} + \frac{1}{2}\hat{\sigma}_P^2 , \tag{4.2.7}$$

$$\hat{\sigma}_P = \frac{\hat{\phi}_1}{\sqrt{\Delta t}} . \tag{4.2.8}$$

Similarly, based on the equation (4.2.5), the parameters μ_C, σ_C can be estimated by the processes

$$\Delta c_j = c_{j+\Delta t} - c_j = \left(\mu_C - \frac{1}{2}\sigma_C^2 \right)\Delta t + \varepsilon'_{j+\Delta t} ,$$

$$\Delta c_j = c_{j+1} - c_j = v_2 + \varepsilon'_{j+1},$$

where the errors are $\varepsilon'_{j+\Delta t} \sim N\left(0, \sigma_C^2 \Delta t\right)$, $\varepsilon'_{j+1} \sim N\left(0, \phi_2^2\right)$, and $c_j = \ln C_j$. Here, v_2 is the sample mean of the array Δc_j, ϕ_2 is the sample standard deviation of the array Δc_j. Then the estimated values of μ_C, σ_C can be calculated as follows

$$\hat{\mu}_C = \frac{\hat{v}_2}{\Delta t} + \frac{1}{2}\hat{\sigma}_C^2, \tag{4.2.9}$$

$$\hat{\sigma}_C = \frac{\hat{\phi}_2}{\sqrt{\Delta t}}. \tag{4.2.10}$$

Here, $\hat{v}_2$ and $\hat{\phi}_2$ are the estimated values of v_2 and ϕ_2.

By equation (4.2.4) and equation (4.2.5), it is easy to get these results,

$$E_t\left[d \ln P\right] = \left(\mu_P - \frac{1}{2}\sigma_P^2\right) dt,$$

$$Var\left[d \ln P\right] = \sigma_P^2 dt,$$

$$E_t\left[d \ln C\right] = \left(\mu_C - \frac{1}{2}\sigma_C^2\right) dt,$$

$$Var\left[d \ln C\right] = \sigma_C^2 dt,$$

$$d \ln P \cdot d \ln C = \sigma_P \sigma_C \rho dt.$$

Since the covariance of $d \ln P$ and $d \ln C$ can be calculated as

$$Cov\left(d \ln P, d \ln C\right) = E\left[d \ln P \cdot d \ln C\right] - E\left[d \ln P\right] E\left[d \ln C\right] = \sigma_P \sigma_C \rho dt,$$

thus, the correlation coefficient estimator $\hat{\rho}_{pc}$ of array Δp_j and array Δc_j can be derived by the definition of correlation coefficient

$$\hat{\rho}_{PC} = \frac{Cov\left(d \ln P, d \ln C\right)}{\sqrt{Var\left(d \ln P\right) Var\left(d \ln C\right)}} = \frac{\sigma_P \sigma_C \rho dt}{\sqrt{\sigma_P^2 dt \sigma_C^2 dt}} = \rho.$$

That is,

$$\hat{\rho} = \hat{\rho}_{pc}, \tag{4.2.11}$$

where $\hat{\rho}$ is the estimated value of the correlation coefficient ρ.

4.3 Dynamic Programming Model

After analyzing the differences of the unit revenue and the unit expenditure between government and investor perspectives, this subsection will discuss the value function of the completed project first, then derive the option value function in each scenario.

4.3.1 Government and Investor Perspectives

By assumption, the government pays the subsidy S to the investors. Meanwhile, only the government needs to pay for the carbon emission cost. The unit revenue and the unit expenditure under government and investor perspectives can be shown in the next two cases. By assumption, the unit revenue P is considered as a whole, which contains all the parts of income, such as the stochastic variable P^g and some related non-stochastic parameters P^x and P^l. So does the unit expenditure C, which contains the stochastic variable P^c and other non-stochastic parameters P^z, and so on.

Case 1: under government perspective

Under the government perspective, the unit revenue P contains the gasoline price P^g, the xylitol price P^x and the pure lignin price P^l. The unit expenditure C includes the corn cob price P^c, the zymin expense P^z, the CO_2 emission cost P^{cb} and the subsidy S. Based on Table 1.1 in Chapter I, the unit revenue P and the unit expenditure C satisfy the following,

$$P = 0.9111P^g + \frac{4}{5}P^x + \frac{2}{3}P^l ,$$

$$C = \frac{20}{3}P^c + P^z + S + 3.15P^{cb} .$$

Based on Table 3.3, the non-stochastic variables are $P^g = 23{,}000$, $P^l = 4{,}500$, $P^z = 2{,}600$, $S = 800$, $P^{cb} = 50$. Using the data of the gasoline price and corn cob price from 1 January, 2012 to 31 May, 2015, the estimated values $\hat{\mu}_P$, $\hat{\sigma}_P$, $\hat{\mu}_C$, $\hat{\sigma}_C$, $\hat{\rho}$ under government perspective are shown in the second column in Table 4.1.

Case 2: under investor perspective

Besides the gasoline price P^g , the xylitol price P^x and the pure lignin price P^l , the subsidy S is one part of the unit revenue P under investor perspective. Only the corn cob price P^c and the zymin price P^z are considered as parts of the unit expenditure C . Thus, the unit revenue and unit expenditure under investor perspective are shown as follows

$$P = 0.9111P^g + \frac{4}{5}P^x + \frac{2}{3}P^l + S ,$$

$$C = \frac{20}{3}P^c + P^z .$$

Using the same data, these estimated values $\hat{\mu}_P$, $\hat{\sigma}_P$, $\hat{\mu}_C$, $\hat{\sigma}_C$, $\hat{\rho}$ under investor perspective can be found in the third column in Table 4.1.

Table 4.1 The parameter estimated values.

parameter	estimated value	
	government	investors
$\hat{\mu}_P$	-0.01192141	-0.01161337
$\hat{\sigma}_P$	0.036685287	0.035737192
$\hat{\mu}_C$	-0.065021634	-0.074469005
$\hat{\sigma}_C$	0.255425363	0.304436916
$\hat{\rho}$	-0.003595696	-0.003539494

Although there exist differences about the unit revenue and the unit expenditure under government and investor perspectives, the processes of solving the option value functions are essentially the same. Substituting the related estimated values under each perspective, the exact results can be obtained.

4.3.2 Value Function of the Completed Project

As a common approach in real option analysis developed by Dixit and Pindyck (1994), dynamic programming can be used to determine the value of the completed project $V(P,C)$ in the infinite time horizon case. Here, the unit revenue P and the unit expenditure C follow GBM as equation (4.2.1) and equation (4.2.2).

According to the Bellman equation

$$rVdt = \pi dt + E_t[dV], \tag{4.3.1}$$

where $E_t[\cdots]$ is the conditional expectation at the current time t, r is the discounted rate, and $\pi(P,C)$ is the instantaneous profit cash flow that satisfies

$$\pi\left(P,C\right)=\max\left\{P-C,0\right\}. \tag{4.3.2}$$

The Bellman equation means that the total value at a small time period dt equals the sum of the instantaneous profit and the increment unit value of the completed project at the period.

From Ito's lemma,

$$dV=V_PdP+V_CdC+\frac{1}{2}V_{PP}\left(dP\right)^2+\frac{1}{2}V_{CC}\left(dC\right)^2+V_{PC}dPdC$$

$$=\left[\frac{1}{2}\sigma_P^2P^2V_{PP}+\frac{1}{2}\sigma_C^2C^2V_{CC}+\rho\sigma_P\sigma_CPCV_{PC}+\mu_PPV_P+\mu_CCV_C\right]dt$$

$$+\mu_PPV_PdB_{1t}+\mu_CCV_CdB_{2t},$$

where $dB_{1t}dt=0$, $dB_{2t}dt=0$, $dB_{1t}dB_{2t}=\rho dt$, $\left(dP\right)^2=\sigma_P^2P^2dt$, $\left(dC\right)^2=\sigma_C^2C^2dt$. Here, V_P and V_C are the first order partial derivatives of the function $V\left(P,C\right)$ with respect to variable P and variable C, V_{PP} and V_{CC} are the second order partial derivatives of the function $V\left(P,C\right)$ with respect to variable P and variable C, V_{PC} is the mixed second order partial derivative of the function $V\left(P,C\right)$ with respect to these variables P and C.

Since $E_t\left[dB_{1t}\right]=0$ and $E_t\left[dB_{2t}\right]=0$, the Bellman equation (4.3.1) can be expressed as a PDE as follows,

$$\frac{1}{2}\sigma_P^2P^2V_{PP}+\frac{1}{2}\sigma_C^2C^2V_{CC}+\rho\sigma_P\sigma_CPCV_{PC}+\mu_PPV_P+\mu_CCV_C-rV+\pi=0, \tag{4.3.3}$$

with the following boundary conditions

$$V\left(0,C\right)=0, \tag{4.3.4}$$

$$V\left(P,\infty\right)=0, \tag{4.3.5}$$

$$V(P,C) \text{ is continuous at } P=C, \tag{4.3.6}$$

$$V_P(P,C),\ V_C(P,C) \text{ are continuous at } P=C. \tag{4.3.7}$$

In the case $P<C$, $\pi(P,C)=0$, the equation (4.3.3) will be

$$\frac{1}{2}\sigma_P^2P^2V_{PP}+\frac{1}{2}\sigma_C^2C^2V_{CC}+\rho\sigma_P\sigma_C PCV_{PC}+\mu_P PV_P+\mu_C CV_C-rV=0. \tag{4.3.8}$$

Obviously, this PDE is a second order elliptic partial differential equation, and the existence of the solution of this kind PDE is ensured by the Cauchy-Kowalevski Theorem (Nakhushev, 2001), which states that the Cauchy problem for any PDE whose coefficients are analytic in the unknown function and its derivatives has a locally unique analytic solution. In order to find the solution of the above PDE, we can use the same idea as in Dixit and Pindyck (1994, p210) . From the economic intuition, if the current values of P and C are doubled, the value of the project will be doubled as well. Thus, this kind of PDE can be solved by reducing to a one variable problem based on the homogeneity of the value function. Let m denote the ratio value $\frac{P}{C}$, then the optimal decision will only depend on the ratio m. In the case $P<C$, the ratio $m=\frac{P}{C}<1$. Correspondingly, the value function is homogeneous of degree 1 in (P,C), that is, the value function can be written as $V(P,C)=Cf(m)$. Here, the function $f(m)$ is unknown.

By the derivation rules,

$$V_P=f'(m),$$

$$V_C=f(m)-mf'(m),$$

$$V_{PP} = \frac{f''(m)}{C},$$

$$V_{PC} = -\frac{mf''(m)}{C},$$

$$V_{CC} = \frac{m^2 f''(m)}{C},$$

the equation (4.3.8) can be changed to an Ordinary Differential Equation (ODE) as follows

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)m^2 f''(m) + (\mu_P - \mu_C)mf'(m) + (\mu_C - r)f(m) = 0 \ . \quad (4.3.9)$$

The characteristic equation of this ODE is

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)\beta(\beta - 1) + (\mu_P - \mu_C)\beta + (\mu_C - r) = 0 \ . \quad (4.3.10)$$

The roots are

$$\beta_1 = \left(\frac{1}{2} - \frac{\mu_P - \mu_C}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu_P - \mu_C}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}\right)^2 - \frac{2(\mu_C - r)}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}},$$

(4.3.11)

$$\beta_2 = \left(\frac{1}{2} - \frac{\mu_P - \mu_C}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu_P - \mu_C}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}\right)^2 - \frac{2(\mu_C - r)}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2}}.$$

(4.3.12)

Suppose that $r > \mu_P > \mu_C$, since

$$\beta_1 + \beta_2 = 1 - \frac{2(\mu_P - \mu_C)}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2} < 1,$$

$$\beta_1\beta_2 = \frac{2(\mu_C - r)}{\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2} < 0,$$

where $\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2 = (\sigma_P - \rho\sigma_C)^2 + (1-\rho)\sigma_C^2 > 0$ by $\sigma_P \in (0,1)$, $\sigma_C \in (0,1)$, $\rho \in (0,1)$, hence, $\beta_1 > 1$, $\beta_2 < 0$.

According to the homogeneity and the boundary condition (4.3.4), the general solution of equation (4.3.9) is just a linear combination of the two power solutions corresponding to these two roots, that is,

$$f(m) = Am^{\beta_1} + Bm^{\beta_2}, \quad m < 1, \quad \beta_1 > 1, \quad \beta_2 < 0.$$

Furthermore, by the boundary conditions (4.3.4) and (4.3.5), the function $f(m)$ satisfies that $f(0) = 0$, $f(m) \to 0$ as $m \to 0$. This requires $B = 0$, so

$$f(m) = Am^{\beta_1}, \quad m < 1, \quad \beta_1 > 1.$$

Thus,

$$V(P,C) = AP^{\beta_1}C^{1-\beta_1}, \quad \frac{P}{C} < 1, \quad \beta_1 > 1. \tag{4.3.13}$$

In the case $P \geq C$, the equation (4.3.3) will be

$$\frac{1}{2}\sigma_P^2 P^2 V_{PP} + \frac{1}{2}\sigma_C^2 C^2 V_{CC} + \rho\sigma_P\sigma_C PCV_{PC} + \mu_P PV_P + \mu_C CV_C - rV + P - C = 0. \tag{4.3.14}$$

Using the same method, the PDE (4.3.14) will be changed to a new ODE as follows

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)m^2 f''(m) + (\mu_P - \mu_C)mf'(m) + (\mu_C - r)f(m) + m - 1 = 0. \tag{4.3.15}$$

Here, the ratio $m = \frac{P}{C} \geq 1$. Since the equation (4.3.15) has the same characteristic equation as equation (4.3.9), so the roots β_1, β_2 are the same as equations (4.3.11) and (4.3.12). Thus, the general solution of the inhomogeneous ordinary equation has two parts: one is the combination of the power solutions of the homogeneous part, another is one particular solution of the inhomogeneous equation.

In fact, the unit revenue can not go to infinite, so that $V(P,C)\to 0$ as $P\to\infty$, that is, $f(m)\to 0$ as $m\to\infty$. Meanwhile, the unknown function $f(m)$ also satisfies the conditions $f(0)=0$, $f(m)\to 0$ as $m\to 0$. Hence, the solution of the homogeneous part $f(m)=Dm^{\beta_1}+Em^{\beta_2}$ satisfies $D=0$, so that the solution will be

$$f(m)=Em^{\beta_2},\ \ m\ge 1,\ \ \beta_2<0.$$

Clearly, $\dfrac{m}{r-\mu_P}-\dfrac{1}{r-\mu_C}$ is a particular solution of the ODE (4.3.15) , thus the solution can be written as

$$f(m)=Em^{\beta_2}+\frac{m}{r-\mu_P}-\frac{1}{r-\mu_C},\ \ m\ge 1,\ \ \beta_2<0.$$

That is,

$$V(P,C)=EP^{\beta_2}C^{1-\beta_2}+\frac{P}{r-\mu_P}-\frac{C}{r-\mu_C},\ \ \frac{P}{C}\ge 1,\ \ \beta_2<0. \tag{4.3.16}$$

Following the equation (4.3.13) and equation (4.3.16), the value function of the completed project is

$$V(P,C)=\begin{cases} AP^{\beta_1}C^{1-\beta_1}, & \dfrac{P}{C}<1 \\ EP^{\beta_2}C^{1-\beta_2}+\dfrac{P}{r-\mu_P}-\dfrac{C}{r-\mu_C}, & \dfrac{P}{C}\ge 1 \end{cases} \tag{4.3.17}$$

where $\beta_1>1$, $\beta_2<0$.

According to the continuity of $V(P,C)$, $V_P(P,C)$, $V_C(P,C)$ at $P=C$, then

$$A = E + \frac{1}{r-\mu_P} - \frac{1}{r-\mu_C},$$

$$A\beta_1 = E\beta_2 + \frac{1}{r-\mu_P},$$

$$A(1-\beta_1) = E(1-\beta_2) - \frac{1}{r-\mu_C},$$

the coefficient parameters must satisfy

$$A = \frac{1}{\beta_1-\beta_2}\left(\frac{1-\beta_2}{r-\mu_P} - \frac{\beta_2}{r-\mu_C}\right), \tag{4.3.18}$$

$$E = \frac{1}{\beta_1-\beta_2}\left(\frac{1-\beta_1}{r-\mu_P} - \frac{\beta_1}{r-\mu_C}\right). \tag{4.3.19}$$

It is easy to prove that $A>0$, $E<0$.

After substituting the estimated values from Table 4.1, the resulting the parameters β_1, β_2, A, E under government and investor perspectives are shown in Table 4.2.

Table 4.2 The parameters values in the function $V(P,C)$.

parameter	government	investors
β_1	1.4352	1.3458
β_2	-2.0284	-1.6826
A	25.9436	25.5293
E	-7.1313	-6.7918

4.3.3 Scenario Option Value Functions

Based on the basic assumption of the two construction stages, the real option model contains three cases - both stages have been completed, only the stage-1 has been completed and the project is not started, as shown in Figure 3.5 in Chapter III. Here, this subsection also uses scenario 0, scenario 1 and scenario 2 to present these three cases sequentially.

Case 1: the option value at scenario 0.

At scenario 0, all the construction stages have been completed, the owners do not need to make any decisions, the option value is the difference of the completed project value and the construction cost, so the option value $F0(P,C)$ at scenario 0 satisfies the following Bellman equation,

$$rF0dt = r\left(V - \frac{J_2}{Q} - \frac{J_1}{Q}\right)dt \,. \tag{4.3.20}$$

In fact, it is easy to obtain the analytic form of the option value function $F0(P,C)$,

$$F0(P,C) = \begin{cases} AP^{\beta_1}C^{1-\beta_1} - \frac{J_2}{Q} - \frac{J_1}{Q}, & \frac{P}{C} < 1 \\ EP^{\beta_2}C^{1-\beta_2} + \frac{P}{r-\mu_P} - \frac{C}{r-\mu_C} - \frac{J_2}{Q} - \frac{J_1}{Q}, & \frac{P}{C} \geq 1 \end{cases} . \tag{4.3.21}$$

Here, the parameters β_1, β_2, A, E are the same as in Table 4.2.

Case 2: the option value at scenario 1.

If only the stage-1 construction has been completed, the owners will face two choices: invest immediately or continue to wait. If the owners choose to invest immediately, they must pay the cost of stage-2 construction. On the other hand, if they continue to wait for some new information and make the decisions later, they only have the stage-1 construction completed. Obviously, the owners will choose the action that can yield the maximum profit. Thus the Bellman equation of the option value $F1(P,C)$ at scenario 1 will be

$$rF1dt = \max\left\{ r\left(V - \frac{J_2}{Q}\right)dt, E_t\left[dF1\right] \right\}. \tag{4.3.22}$$

In the "wait" or "continue" region, based on $rF1dt = E_t\left[dF1\right]$, the PED (4.3.23) can easily be obtained by Ito's lemma and the conditional expectation

$$\frac{1}{2}\sigma_P^2 P^2 F1_{PP} + \frac{1}{2}\sigma_C^2 C^2 F1_{CC} + \rho\sigma_P\sigma_C PCF1_{PC} + \mu_P PF1_P + \mu_C CF1_C - rF1 = 0 \text{ ,} \tag{4.3.23}$$

with boundary conditions

$$F1(0,C) = 0, \tag{4.3.24}$$

$$F1(P,\infty) = 0, \tag{4.3.25}$$

$$F1\left(P_1^*, C_1^*\right) = V\left(P_1^*, C_1^*\right) - \frac{J_2}{Q}, \tag{4.3.26}$$

$$F1_P\left(P_1^*, C_1^*\right) = V_P\left(P_1^*, C_1^*\right), \tag{4.3.27}$$

$$F1_C\left(P_1^*, C_1^*\right) = V_C\left(P_1^*, C_1^*\right). \tag{4.3.28}$$

Here, P_1^*, C_1^* are the threshold values at scenario 1. Equation (4.3.26) is the value-matching condition, the next two equations (4.3.27) and (4.3.28) are the smooth-pasting conditions.

Using the same method as Section 4.3.2, let ratio $m = \frac{P}{C}$, then the option value function $F1(P,C)$ can be written as $F1(P,C) = Cf_1(m)$, where the function $f_1(m)$ is also unknown, so the equation (4.3.23) will be changed to an ODE as follows

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_P\sigma_C + \sigma_C^2\right)m^2 f_1''(m) + \left(\mu_P - \mu_C\right)m f_1'(m) + \left(\mu_C - r\right)f_1(m) = 0 \ . \quad (4.3.29)$$

Similarly, the solution has the form $f_1(m) = A_1 m^{\beta_1} + B_1 m^{\beta_2}$, here β_1, β_2 are the same as (4.3.11) and (4.3.12). By the boundary conditions (4.3.24) and (4.3.25), the function $f_1(m)$ satisfies $f_1(0) = 0$, $f_1(m) \to 0$ as $m \to 0$, that requires $B_1 = 0$. Thus, the solution is $f_1(m) = A_1 m^{\beta_1}$, $\beta_1 > 1$. That is,

$$F1(P,C) = A_1 P^{\beta_1} C^{1-\beta_1}, \ \beta_1 > 1. \quad (4.3.30)$$

If the threshold values satisfy $P_1^* < C_1^*$, according to the smooth-pasting conditions (4.3.27) and (4.3.28), we have

$$A_1\beta_1 P_1^{*\beta_1 - 1} C_1^{*1-\beta_1} = A\beta_1 P_1^{*\beta_1 - 1} C_1^{*1-\beta_1} \ ,$$

$$A_1 P_1^{*\beta_1}\left(1-\beta_1\right) C_1^{*-\beta_1} = A P_1^{*\beta_1}\left(1-\beta_1\right) C_1^{*-\beta_1} \ ,$$

then $A_1 = A$, which is contradicted by the value-matching condition (4.3.26),

$$A_1 P_1^{*\beta_1} C_1^{*1-\beta_1} = A P_1^{*\beta_1} C_1^{*1-\beta_1} - \frac{J_2}{Q}.$$

Hence, the threshold values P_1^*, C_1^* satisfy $P_1^* \geq C_1^*$.

By the value-matching condition (4.3.26), the smooth-pasting conditions (4.3.27) and (4.3.28), then

$$A_1 P_1^{*\beta_1} C_1^{*1-\beta_1} = E P_1^{*\beta_2} C_1^{*1-\beta_2} + \frac{P_1^*}{r-\mu_P} - \frac{C_1^*}{r-\mu_C} - \frac{J_2}{Q} \ ,$$

$$A_1\beta_1 P_1^{*\beta_1 - 1} C_1^{*1-\beta_1} = E\beta_2 P_1^{*\beta_2 - 1} C_1^{*1-\beta_2} + \frac{1}{r-\mu_P} \ ,$$

$$A_1 P_1^{*\beta_1}\left(1-\beta_1\right) C_1^{*-\beta_1} = E P_1^{*\beta_2}\left(1-\beta_2\right) C_1^{*-\beta_2} - \frac{1}{r-\mu_C} \ .$$

By the assumption on the ratio m, suppose that $m_1^* = \frac{P_1^*}{C_1^*}$ is the threshold value of the ratio $m = \frac{P}{C}$, then $m_1^* \geq 1$. These three equations can be rewritten as

$$A_1 m_1^{*\beta_1} = E m_1^{*\beta_2} + \frac{m_1^*}{r-\mu_P} - \frac{1}{r-\mu_C} - \frac{1}{C_1^*}\frac{J_2}{Q},$$

$$A_1 \beta_1 m_1^{*\beta_1 - 1} = E\beta_2 m_1^{*\beta_2 - 1} + \frac{1}{r-\mu_P},$$

$$A_1(1-\beta_1) m_1^{*\beta_1} = E(1-\beta_2) m_1^{*\beta_2} - \frac{1}{r-\mu_C}.$$

From the second and the third equations, it is easy to find that m_1^* is the root of the following equation

$$E(\beta_1 - \beta_2) m^{\beta_2} + (\beta_1 - 1)\frac{m}{r-\mu_P} - \beta_1 \frac{1}{r-\mu_C} = 0 . \tag{4.3.31}$$

Let $g(m)$ denote the left part of the equation (4.3.31), then

$$g'(m) = E(\beta_1 - \beta_2)\beta_2 m^{\beta_2 - 1} + (\beta_1 - 1)\frac{1}{r-\mu_P}.$$

Since $E < 0$, $\beta_1 > 1$, $\beta_2 < 0$, $r > \mu_P > \mu_C$, then $g'(m) > 0$ for all $m \geq 1$. Hence, $g(m)$ is an increasing function in the interval $[0,\infty)$. Clearly, because of $g(0) = -\beta_1 \frac{1}{r-\mu_C} < 0$, $g(m) > 0$ as long as m is large enough. Thus, the equation (4.3.31) has only one root, this conclusion will be shown again in Section 4.4.

Furthermore,

$$A_1 = E\frac{\beta_2}{\beta_1} m_1^{*\beta_2 - \beta_1} + \frac{1}{\beta_1}\frac{1}{r-\mu_P} m_1^{*1-\beta_1}, \tag{4.3.32}$$

$$C_1^* = \frac{\frac{J_2}{Q}}{-A_1 m_1^{*\beta_1} + E m_1^{*\beta_2} + \frac{m_1^*}{r-\mu_P} - \frac{1}{r-\mu_C}}, \tag{4.3.33}$$

$$P_1^* = \frac{m_1^* \frac{J_2}{Q}}{-A_1 m_1^{*\beta_1} + E m_1^{*\beta_2} + \frac{m_1^*}{r-\mu_P} - \frac{1}{r-\mu_C}}. \tag{4.3.34}$$

Hence,

$$F1(P,C) = \begin{cases} A_1 P^{\beta_1} C^{1-\beta_1}, & \frac{P}{C} < \frac{P_1^*}{C_1^*} \\ E P^{\beta_2} C^{1-\beta_2} + \frac{P}{r-\mu_P} - \frac{C}{r-\mu_C} - \frac{J_2}{Q}, & \frac{P}{C} \geq \frac{P_1^*}{C_1^*} \geq 1 \end{cases}. \tag{4.3.35}$$

All these parameters results m_1^*, A_1, C_1^*, P_1^* under government and investor perspectives can be calculated as well.

Table 4.3 The parameters values in the function $F1(P,C)$.

parameter	government	investors
m_1^*	2.0654	2.2548
A_1	12.3874	13.5859
C_1^*	-1.3350e+17	1.8690e+17
P_1^*	-2.7574e+17	4.2142e+17

Case 3: the option value at scenario 2.

If the project is not started, the owners also have two choices: invest in the stage-1 construction immediately or still wait. So the option value function

$F2(P,C)$ at scenario 2 satisfies the following Bellman equation

$$rF2dt = \max\left\{ r\left(F1 - \frac{J_1}{Q}\right)dt, E_t\left[dF2\right] \right\}. \tag{4.3.36}$$

Likewise, $rF2dt = E_t\left[dF2\right]$ presents the "wait" or "continue" region. According to the PDE

$$\frac{1}{2}\sigma_P^2 P^2 F2_{PP} + \frac{1}{2}\sigma_C^2 C^2 F2_{CC} + \rho\sigma_P\sigma_C PCF2_{PC} + \mu_P PF2_P + \mu_C CF2_C - rF2 = 0 , \tag{4.3.37}$$

with boundary conditions

$$F2(0,C) = 0, \tag{4.3.38}$$

$$F2(P,\infty) = 0, \tag{4.3.39}$$

$$F2\left(P_2^*, C_2^*\right) = F1\left(P_2^*, C_2^*\right) - \frac{J_1}{Q}, \tag{4.3.40}$$

$$F2_P\left(P_2^*, C_2^*\right) = F1_P\left(P_2^*, C_2^*\right), \tag{4.3.41}$$

$$F2_C\left(P_2^*, C_2^*\right) = F1_C\left(P_2^*, C_2^*\right). \tag{4.3.42}$$

After a similar discussion process as in Case 2, the general solution of the option value function $F2(P,C)$ is

$$F2(P,C) = \begin{cases} A_2 P^{\beta_1} C^{1-\beta_1}, & \dfrac{P}{C} < \dfrac{P_2^*}{C_2^*} \\ EP^{\beta_2} C^{1-\beta_2} + \dfrac{P}{r-\mu_P} - \dfrac{C}{r-\mu_C} - \dfrac{J_2}{Q} - \dfrac{J_1}{Q}, & \dfrac{P}{C} \geq \dfrac{P_2^*}{C_2^*} \end{cases} \tag{4.3.43}$$

where

$$A_2 = E\frac{\beta_2}{\beta_1} m_2^{*\beta_2 - \beta_1} + \frac{1}{\beta_1}\frac{1}{r-\mu_P} m_2^{*1-\beta_1}, \tag{4.3.44}$$

$$C_2^* = \frac{\dfrac{J_2 + J_1}{Q}}{-A_2 m_2^{*\beta_1} + E m_2^{*\beta_2} + \dfrac{m_2^*}{r - \mu_P} - \dfrac{1}{r - \mu_C}}, \tag{4.3.45}$$

$$P_2^* = \frac{m_2^* \dfrac{J_2 + J_1}{Q}}{-A_2 m_2^{*\beta_1} + E m_2^{*\beta_2} + \dfrac{m_2^*}{r - \mu_P} - \dfrac{1}{r - \mu_C}}. \tag{4.3.46}$$

Here, the threshold ratio value m_2^* is the root of equation (4.3.31) as well, and $m_2^* = \dfrac{P_2^*}{C_2^*} \geq m_1^* \geq 1$.

Furthermore, these parameters values m_2^*, A_2, C_2^*, P_2^* used in function $F2(P, C)$ under government and investor perspectives can be obtained in the same way, and they are shown in Table 4.4.

Table 4.4 The parameters values in the function $F2(P, C)$.

parameter	government	investors
m_2^*	2.0654	2.2548
A_2	12.3874	13.5859
C_2^*	-1.8690e+18	3.7380e+17
P_2^*	-3.8603e+18	4.2142e+17

4.4 Basic Analysis

Based on these option value functions, especially, the threshold values in each case, there appear both similar and distinct phenomena under government and investor perspectives.

4.4.1 Scenarios Analysis

In order to observe the phenomena hidden in these option value functions, these parameters used in the option functions are collected together as shown as Table 4.5 and Table 4.6.

Case 1: government perspective.

From Table 4.5, the threshold ratio value m_1^* equals to m_2^*, which has been proved from point of the mathematical view. It means that the threshold ratio values are the same, whether the government decides to construct the stage-1 or the stage-2. However, the threshold value C_1^* differs from C_2^*, the same is true for P_1^* and P_2^* as well. Obviously, the threshold values C_1^*, P_1^*, C_2^*, P_2^* are less than zero which does not happen in real life. After observing the unit revenue and the unit expenditure, if the ratio value $\frac{P}{C}$ is greater than 2.0654, the government can choose to invest in the stage-1 construction at scenario 2, or stage-2 construction at scenario 1.

Table 4.5 The parameters values under government perspective.

parameter	value	parameter	value	parameter	value
β_1	1.4352	m_1^*	2.0654	m_2^*	2.0654
β_2	-2.0284	A_1	12.3874	A_2	12.3874
A	25.9436	C_1^*	-1.3350e+17	C_2^*	-1.8690e+18
E	-7.1313	P_1^*	-2.7574e+17	P_2^*	-3.8603e+18

Using MATLAB, the figures of the completed project value function $V(P,C)$, the option value functions $F0(P,C)$, $F1(P,C)$, $F2(P,C)$ at each scenario can be shown in Figure 4.1. These four pictures seem very similar. According to these value functions, the government can predict the option value and make the optimal decisions based on the change of the unit revenue and the unit expenditure at any time.

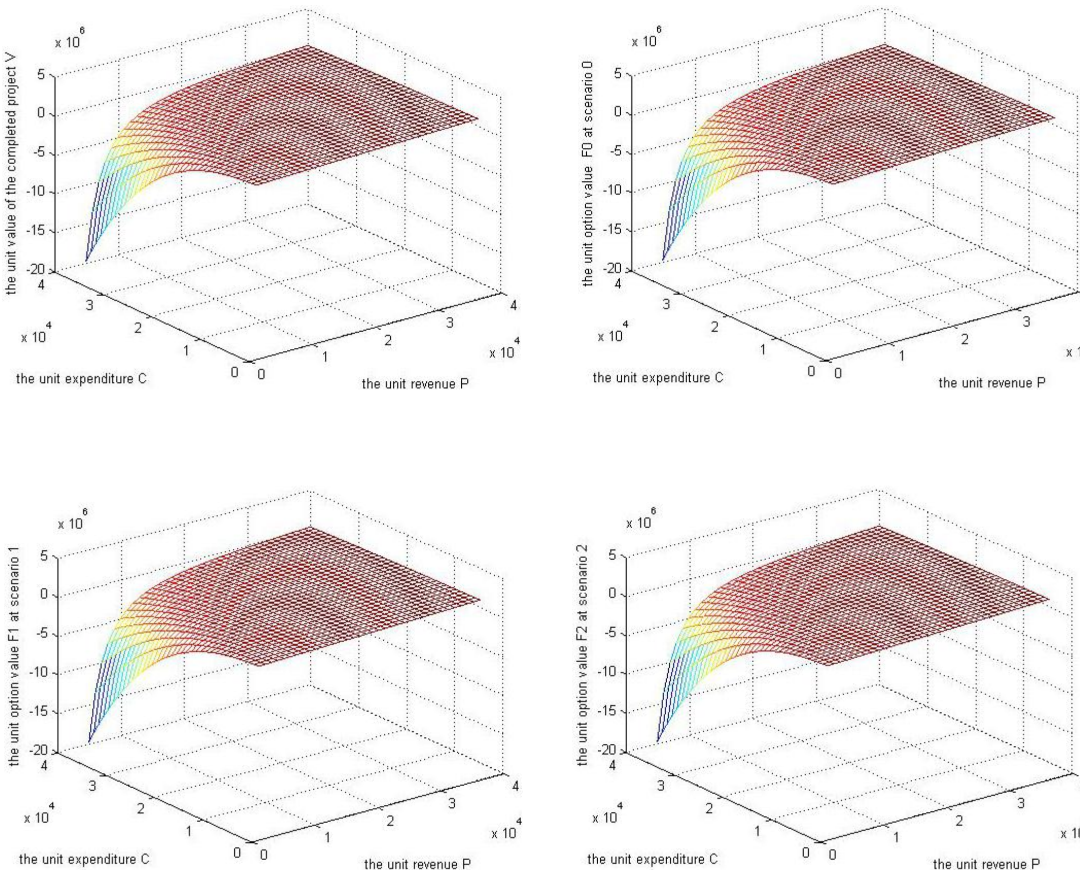


Figure 4.1 The figures of the value functions under government perspective.

Case 2: investor perspective

Table 4.6 shows the parameters values used in these value functions under investor perspective. Clearly, the threshold ratio values m_1^* and m_2^* are the same no matter which stage is constructed by the investors. However, the related threshold values C_1^*, P_1^*, C_2^*, P_2^* are different. Compared with Table 4.5, the threshold values C_1^*, P_1^*, C_2^*, P_2^* are positive under investor perspective. The investors can choose the action "invest" when the ratio value $\frac{P}{C}$ is greater than 2.2548. Although it is similar to Figure 4.1, the figure of each value function in Figure 4.2 is obviously higher than the related one. The reason is that the investors can get the subsidy from the government, and they do not need to pay the carbon emission cost. In brief, the investors get more unit revenue and pay less unit expenditure than the government. Meanwhile, the threshold ratio value m^* (which is ignored the subscript) under investor perspective is greater than it under government perspective.

Table 4.6 The parameters values under investor perspective.

parameter	value	parameter	value	parameter	value
β_1	1.3458	m_1^*	2.2548	m_2^*	2.2548
β_2	-1.6826	A_1	13.5859	A_2	13.5859
A	25.5293	C_1^*	1.8690e+17	C_2^*	3.7380e+17
E	-6.7918	P_1^*	4.2142e+17	P_2^*	8.4285e+17

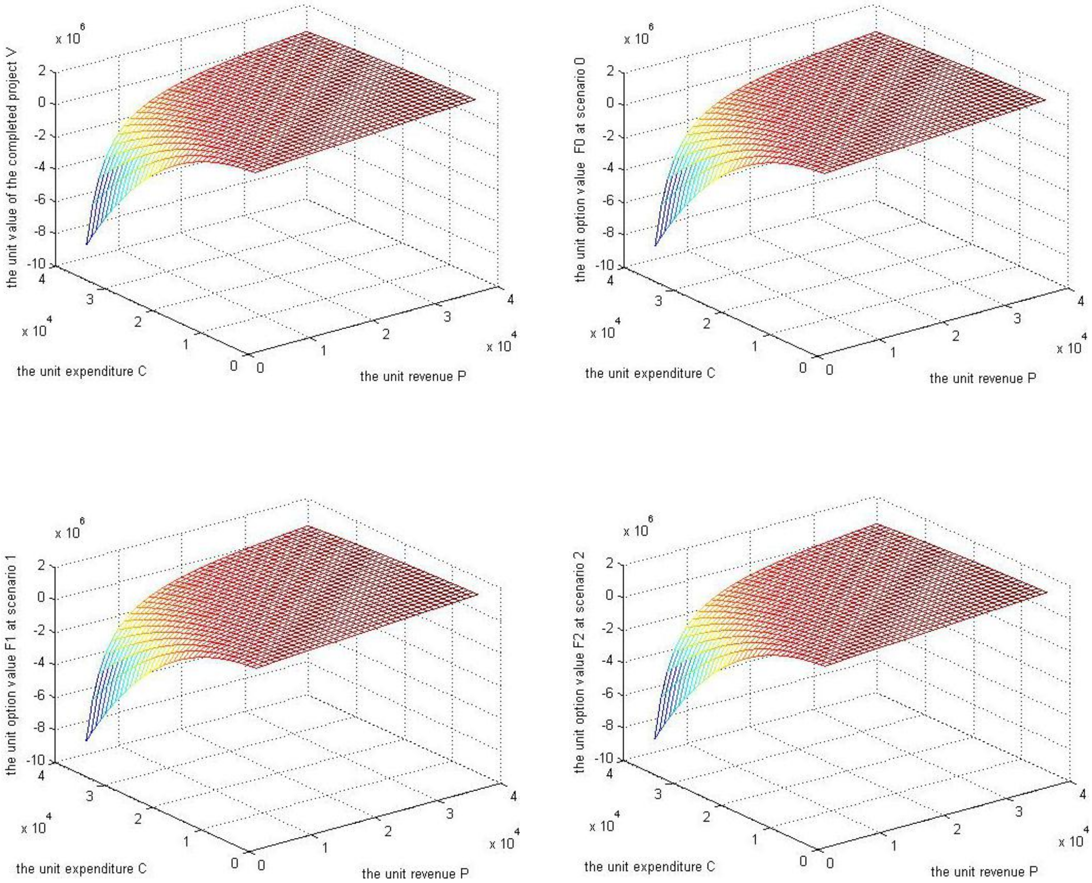


Figure 4.2 The figures of the value functions under investor perspective.

Under both government and investor perspectives, Figure 4.3 shows that all the curves of the history ratio $\frac{P}{C}$ are above the threshold ratio values m^*. It means that these curves are in the “invest” region, since the government pays the subsidy and there are some by-products in the production process. Obviously, the ratio $\frac{P}{C}$ under government perspective is lower than that under investor perspective, because the government pays more expenditure and receives less revenue than the investors. At the current subsidy level, both the government and the investors can choose action “invest” at both scenario 1 and scenario 2 cases, if the cellulosic ethanol project can produce some by-products at the same time.

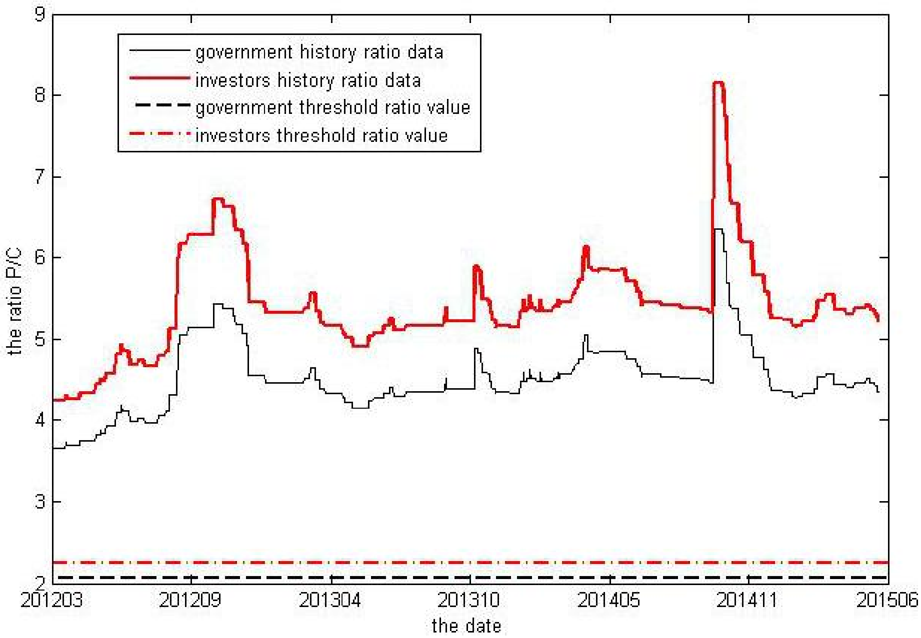


Figure 4.3 The history ratio $\frac{P}{C}$ and the threshold ratio value m^*.

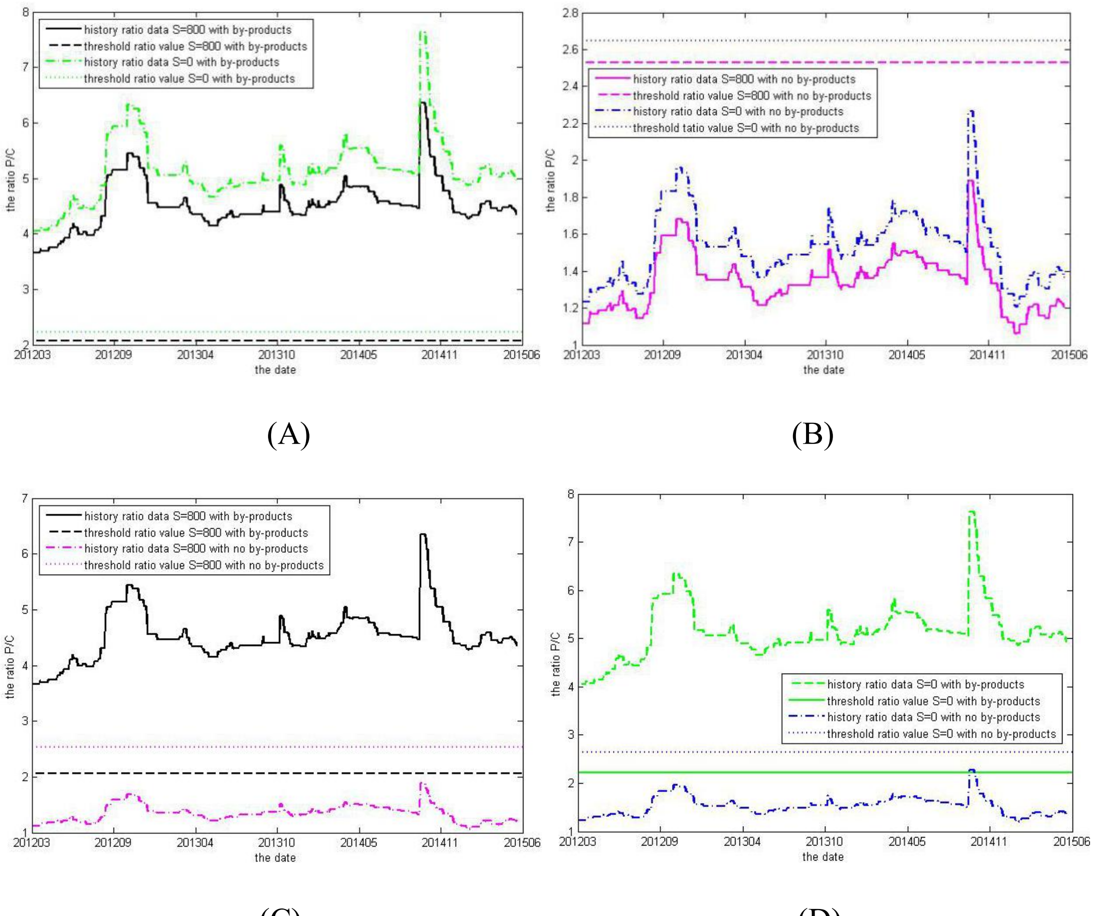
4.4.2 Effects of the Subsidy and the By-products

Besides the basic Case 1 in Table 4.7 and Table 4.8 discussed in Section 4.4.1, this subsection considers another three cases - with subsidy and by-products or not. These results are shown under government and investor perspectives as well.

Table 4.7 The parameters values in different cases under government perspective.

parameter	Case 1	Case 2	Case 3	Case 4
	S=800 with by-products	S=0 with by-products	S=800 with no by-products	S=0 with no by-products
β_1	1.4352	1.3627	1.7617	1.6296
β_2	-2.0284	-1.7368	-1.3479	-1.2408
A	25.9436	25.4544	14.8003	14.8107
E	-7.1313	-6.8618	-9.1916	-8.4225
m_1^*	2.0654	2.2203	2.5276	2.6452
A_1	12.3874	13.2495	4.2264	4.9449
C_1^*	-1.3350e+17	-1.8690e+17	9.3450e+17	3.1150e+17
P_1^*	-2.7574e+17	-4.1497e+17	2.3620e+18	8.2398e+17
m_2^*	2.0654	2.2203	2.5276	2.6452
A_2	12.3874	13.2495	4.2264	4.9449
C_2^*	1.8690e+18	-3.7380e+17	1.8690e+18	6.2300e+17
P_2^*	3.8603e+18	-8.2994e+17	4.7240e+18	1.6480e+18

Comparing Case 1 and Case 2, the values of β_1 and A decrease, but the values of β_2 and E increase. Because of the existence of the by-products in the cellulosic ethanol project, the threshold values C_1^*, P_1^* at scenario 1 and C_2^*, P_2^* at the scenario 2 decrease, the threshold ratio value $m^* = \dfrac{P^*}{C^*}$ increases, even if the government does not pay the subsidy. Similarly, if there does not exist any by-product as Case 3 and Case 4 shown, only β_1 decreases, all the values of β_2, A, E increase. The threshold values C_1^*, P_1^*, C_2^*, P_2^* and the threshold ratio value m^* show the same phenomena with the comparison with Case 1 and Case 2, if the government cancels the subsidy.



(A) (B)

(C) (D)

Figure 4.4 Comparison of two cases under government perspective.

Based on the historical data, Figure 4.4(A) shows that if by-products exist, regardless of whether the government pays the subsidy or not, all the ratios $\frac{P}{C}$ are over the lines of the threshold ratio value m^*. So that the government can invest in each construction stage, even if it must pay the subsidy of 800 yuan. Thus, for the government, choosing action "invest" is optimal. However, if there are no by-products, Figure 4.4(B) shows obvious differences; it is optimal for the government to wait, even if the government cancels the subsidy. That is, the government must clearly consider to invest in the project or not if there do not exist any by-products, or the government chooses to spend more efforts to find some by-products, especially high value by-products.

At the current subsidy level $S = 800$, the effect of by-products seems more important at scenario 2, since the threshold values C_1^*, P_1^* are much large in Case 3. But the changes of the threshold values are small at scenario 1. This conclusion can been seen clearly from Figure 4.4(C). The ratio curve with by-products is over the line of threshold ratio, but another ratio curve with no by-products is under the corresponding line of the threshold ratio. If the government cancels the subsidy, the effect of by-products shows more importance at scenario 1 and scenario 2, since all these differences of each threshold values C_1^*, P_1^*, C_2^*, P_2^* between Case 2 and Case 4 are obvious. This result is shown in Figure 4.4(C) and Figure 4.4(D).

Obviously, the influence of the by-products is more significant than the effect of the subsidy under government perspective. In fact, there exists the same phenomena under investor perspective. Similarly, improving the technology and making full use of the raw materials to find by-products are effective ways to reduce the threshold ratio level to the investors.

Table 4.8 The parameters values in different cases under investor perspective.

parameter	Case 1	Case 2	Case 3	Case 4
	S=800 with by-products	S=0 with by-products	S=800 with no by-products	S=0 with no by-products
β_1	1.3458	1.3486	1.5764	1.6040
β_2	-1.6826	-1.6779	-1.2680	-1.2157
A	25.5293	25.3528	15.6385	14.8028
E	-6.7918	-6.8076	-8.1156	-8.2746
m_1^*	2.2548	2.2579	2.6134	2.6758
A_1	13.5859	13.4303	5.6614	5.0990
C_1^*	1.8690e+17	1.8690e+17	-3.1150e+17	9.3450e+17
P_1^*	4.2142e+17	4.2201e+17	-8.1408e+17	2.5005e+18
m_2^*	2.2548	2.2579	2.6134	2.6758
A_2	13.5859	13.4303	5.6614	5.0990
C_2^*	3.7380e+17	-6.2300e+17	-6.2300e+17	1.8690e+18
P_2^*	8.4285e+17	-1.4067e+18	-1.6282e+18	5.0010e+18

Under investor perspective, based on Case 1 and Case 2 in Table 4.8, the values of β_1 and β_2 increase, but the values of A and E decrease, all these variations are not so obvious. If there exist some by-products in the cellulosic ethanol project, the threshold ratio m^* changes a little at both scenario 1 and scenario 2, even if the government cancels the subsidy. The threshold values C_1^*, P_1^* vary a little at Case 1 and Case 2, but the threshold values C_2^*, P_2^* change from positive to negative obviously. Compared with Case 3 and Case 4, if there are no by-products, the change of the threshold ratio m^* is not so large, whether the government pays the subsidy to the investors or not. On the other hand, the negative threshold values C_1^*, P_1^*, C_2^*, P_2^* become positive.

As Figure 4.5(A) shows, regardless of the government paying the subsidy or not, the curves of the history ratio $\frac{P}{C}$ are close to another, and the same is true for the lines of the threshold ratio m^*. Due to the important role of the by-products, similarly to Figure 5.4(A), all these curves of the history ratio $\frac{P}{C}$ are over the lines of the threshold ratio value m^*. Thus the investors can choose the optimal action "invest", even if the government does not pay any subsidy. Figure 4.5(B) shows an obvious difference when there do not exist any by-products. Since all these history ratios $\frac{P}{C}$ are under the line of the threshold value m^*, so that the investors can choose the optimal action "wait" and observe the movements of the unit revenue and the unit expenditure, even if the government pays the subsidy.

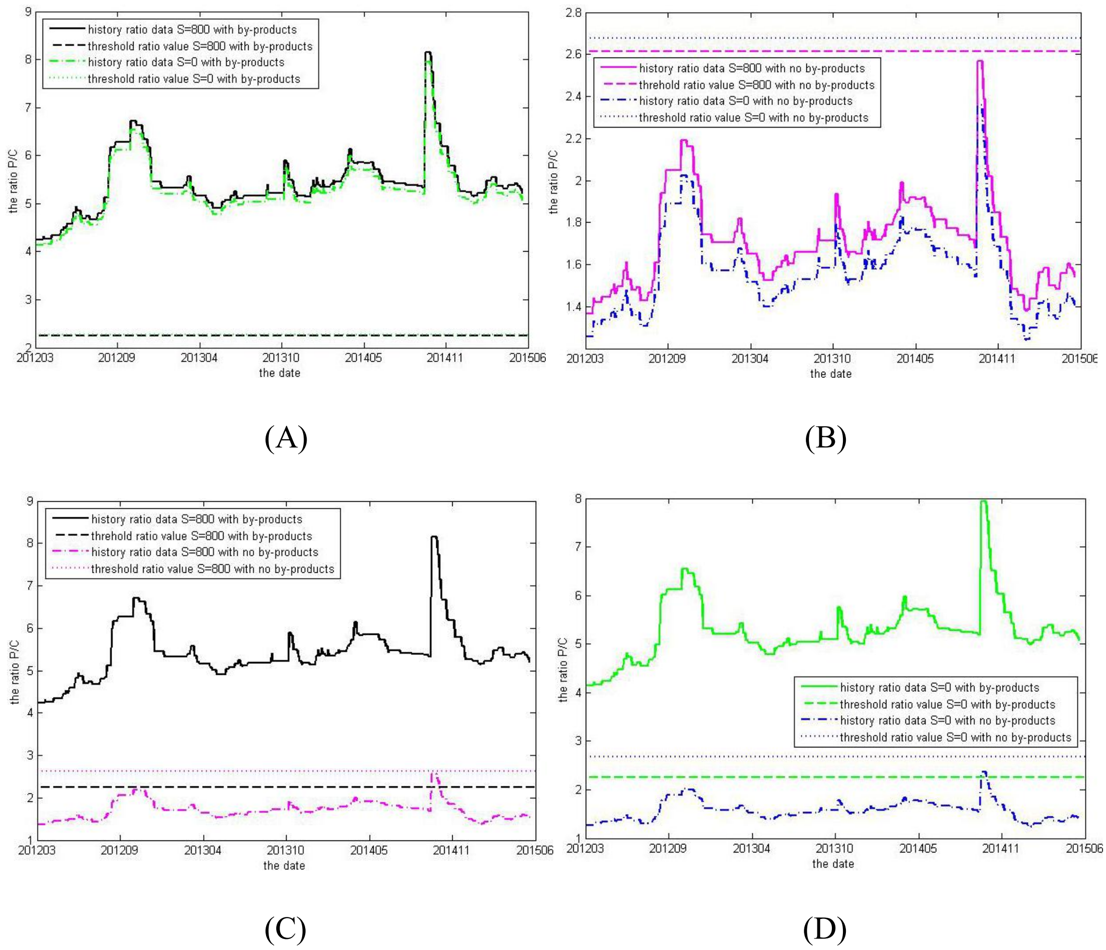


Figure 4.5 Comparison of two cases under investor perspective.

The comparison result from Case 1 and Case 3 also shows the important role of the by-products, if the investors can obtain the current subsidy $S = 800$. The reason is that the threshold ratio value m^* increases, but the threshold values C_1^*, P_1^* and C_2^*, P_2^* decrease obviously. This conclusion can be seen clearly in Figure 4.5(C). The history ratio curve with by-products is higher than the related threshold ratio line, but another history ratio curve with no by-products is under the corresponding threshold ratio line. This phenomenon is similar to that of the subsidy $S = 0$ under government perspective.

If the government cancels the subsidy, the threshold ratio value m^* increases as well. Specially, the effect of by-products is much bigger at scenario 2, since the differences of the threshold values C_2^*, P_2^* are more obviously than that of C_1^*, P_1^*. The same phenomenon is also shown in Figure 4.4(D).

By the discussion with the views of the government and the investors, it is not difficult to find the common point, that improving the technology to find more by-products is the most effective way to reduce the threshold ratio level under investor perspective as well.

4.4.3 Effects of the Proportion of Stage-1 Construction Cost

In the real world, it is necessary to consider the multistage construction investment problem, since every investment stage may usually require different technical or managerial skills. Meanwhile, the owners are more likely to complete the early stages and wait to make decisions about proceeding with the later stages after observing the change of situation. Thus the proportion of every stage may affect the option values in the real option model.

By these assumptions, although $J_1 = aC_{other}$ is an increasing function and $J_2 = (1-a)C_{other}$ is a decreasing function of the variable a, the value function $V(P,C)$ has no relation with the variable a. If all the construction stages have already been completed, that is, the construction cost for each stage has been expended, the option value function $F0(P,C)$ is unaffected by the variable a as well. If only the stage-1 construction has been completed, although the parameter A_1 and the threshold ratio m_1^* are independent with the ratio a, the threshold values

C_1^* and P_1^* will decrease with the proportion a increasing. The option value $F1(P,C)$ will increase with the proportion a increasing when $\frac{P}{C} \geq m_1^*$. For both government and investors, they will get more revenues if they invest the stage-2 construction. Differently at the scenario 2, all the parameters A_2, m_2^*, C_2^* and P_2^* are independent of the proportion a, so is the option value function $F2(P,C)$. That is, no matter how the proportion of the stage-1 construction cost changes, the option value does not change when the project is not started .

Different from the discussion in Chapter III, besides the beginning time, the effect of the proportion of stage-1 construction cost can be discussed at any time so long as the information about the unit revenue and the unit expenditure are known.

CHAPTER V

CONCLUSIONS

Based on the basic analysis of the lattice tree and dynamic programming models that have been shown separately in Chapter III and Chapter IV, Chapter V will discuss further about the conclusions, by comparing these two models.

5.1 Comparison Results

The multistage evaluation model with double stochastic variables based on lattice tree method is of discrete type. It shows the main conclusions. Firstly, at the current subsidy level, if the stage-1 construction has been completed, both government and investors can get higher decision value than in other scenarios, although the government must pay the carbon emission cost and the subsidy. Secondly, because of the high value by-products, the decision values at each scenario are positive. With the subsidy increasing, the decision value decreases under government perspective but increases under investor perspective. Meanwhile these two decision value curves intersect at a subsidy 800 yuan. Thirdly, adding the cost of the stage-1 construction can enhance the benefit of the cellulosic ethanol project at scenario 1, and it gives more influence to investors. At last, if there do not exist any by-products, then the decision values are negative if both construction stages have been completed for both government and investors. Meanwhile, the decision value increases obviously if only the stage-1 construction has been completed. Thus, reducing the subsidy can ease the loss of the government and cut down the benefit of the investors.

The time horizon in the lattice tree model is usually finite. However, the time horizon in dynamic programming model is infinite. Firstly, for both government and investors, the threshold ratio values are the same at scenario 1 and scenario 2, but the threshold values show differences at these scenarios. Secondly, although the government pays the carbon emission cost and the subsidy, the value functions seem very similar. Thirdly, with the subsidy increasing, the decision value decreases under government perspective but increases under investor perspective as well. Fourthly, if there do not exist any by-products in the cellulosic ethanol project, the subsidy will affect the action chosen by the government and investors. If there exist some by-products, the influence of the subsidy is not very obvious. The project can get more revenue from selling the by-products at the same time. At last, adding the cost of the stage-1 construction can enhance the option value of the cellulosic ethanol project if the stage-1 construction has been completed.

In sum, improving the technology and making full use of the raw materials to find more high value by-products are effective ways to enhance the revenues of a cellulosic ethanol plant.

Comparing with these two models, if we only consider two cases - up and down - about the movements of the unit revenue and the unit expenditure, the option values at each scenario can be obtained by the related option value functions, and these results can be shown as quadrinomial lattice tree. Then the government or the investors can make the optimal decisions of the project following the lattice tree model. Although the lattice tree method is easily understandable and implemented, there are still more uncertainties about the stochastic variables in real life. So that it is difficult to handle the uncertainty, as the tree is expanding exponentially with the stochastic factors increasing.

However, the time in dynamic programming method can be considered as either discrete or continuous. Based on the infinite time horizon case, the continuous option value functions in dynamic programming model will help the government or the investors to make decisions at any time if they have the information about the the unit revenue and the unit expenditure no matter how they change.

The empirical results indicate that the dynamic programming model is better to predict the past expansionary behavior. In addition, the multistage construction and double stochastic variables model can effectively used to be evaluate the influence of policy effects to revenues and expenditures.

5.2 Future Research

Although this thesis has done some research about the influence of renewable energy policy for Chinese cellulosic ethanol plants with two real option approaches, it still merits further thinking and improving. Considering that the GBM has some limitations in application, we can discuss some more complex stochastic processes, such as mean-reverting process or jump process. The second point that can be improved is the parameters used in these models, since most parameters are considered as constants. We can consider more stochastic variables, besides gasoline price and corn cob price, the prices of by-products and raw material zymin may fluctuate with time elapsed as well. Or in real life, the drift and volatility may change with the varying market information, it is also necessary to consider them as stochastic variables.

Without doubt, all these ideas may lead to more complex lattice tree or stochastic differential equation, which will be hard to solve. All these works will be carried out in the future.

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APPENDICES

APPENDIX A

THE LIMITATION OF THE EUROPEAN CRR MODEL

Based on the third step in the binomial tree, the option price at time t_j can be calculated by discounting the conditional expectation of the option price at time t_{j+1}, that is,

$$V_j^k = E\left[e^{-r\Delta t}V_{j+1} \middle| V_0, V_1, \ldots, V_j\right] = e^{-r\Delta t}\left[pV_{j+1}^k + (1-p)V_{j+1}^{k+1}\right].$$

Similarly, $V_j^{k+1} = e^{-r\Delta t}\left[pV_{j+1}^{k+1} + (1-p)V_{j+1}^{k+2}\right]$. Here, $k = 0,1,\ldots,j$ and $0 \le j \le N$, k is the number of the price downward movements, $\Delta t = \dfrac{T}{N}$, N is a the positive integer, T is the expiration date of the option, r is the risk-free interest rate, $p = \dfrac{e^{r\Delta t} - d}{u - d}$ is the risk-neutral probability of the asset price increase. So that

$$\begin{aligned}
V_{j-1}^k &= e^{-r\Delta t}\left[pV_j^k + (1-p)V_j^{k+1}\right] \\
&= e^{-r\Delta t}\left[p\left(e^{-r\Delta t}\left[pV_{j+1}^k + (1-p)V_{j+1}^{k+1}\right]\right) + (1-p)\left(e^{-r\Delta t}\left[pV_{j+1}^{k+1} + (1-p)V_{j+1}^{k+2}\right]\right)\right] \\
&= e^{-2r\Delta t}\left[p^2V_{j+1}^k + 2p(1-p)V_{j+1}^{k+1} + (1-p)^2V_{j+1}^{k+2}\right].
\end{aligned}$$

By induction method, we have

$$V_0 = e^{-Nr\Delta t}\sum_{k=0}^{N}\binom{N}{k}p^{N-k}(1-p)^kV_N^k = e^{-rT}\sum_{k=0}^{N}\binom{N}{k}p^{N-k}(1-p)^kV_N^k.$$

Taking European call option as an example, at the expiration date $T = t_N$,

$V_N^k = \max\left\{S_N^k - K, 0\right\}$. That is, $V_N^k = \max\left\{S_0 u^{N-k} d^k - K, 0\right\} = \left(S_0 u^{N-k} d^k - K, 0\right)^+$, since the set of possible prices at time t_j is $S_j^k = S_0 u^{j-k} d^k$, where $u = e^{\sigma\sqrt{\Delta t}}$ is the range of the price upward movements, $d = \dfrac{1}{u} = e^{-\sigma\sqrt{\Delta t}}$ is the range of the price downward movements.

Hence,

$$V_0 = e^{-rT} \sum_{k=0}^{N} \binom{N}{k} p^{N-k} (1-p)^k \left(S_0 u^{N-k} d^k - K, 0\right)^+ .$$

Let $m = \min\left\{0 \le k \le N : S_0 u^{N-k} d^k - K \ge 0\right\}$, then

$$V_0 = e^{-rT} S_0 \sum_{k=m}^{N} \binom{N}{k} p^{N-k} (1-p)^k u^{N-k} d^k - e^{-rT} K \sum_{k=m}^{N} \binom{N}{k} p^{N-k} (1-p)^k$$

$$= e^{-rT} S_0 \sum_{k=m}^{N} \binom{N}{k} (up)^{N-k} \left(d(1-p)\right)^k - e^{-rT} K \sum_{k=m}^{N} \binom{N}{k} p^{N-k} (1-p)^k .$$

According to the Central Limit Theory,

$$\lim_{N\to\infty} e^{-rT} \sum_{k=m}^{N} \binom{N}{k} (up)^{N-k} \left(d(1-p)\right)^k = \lim_{N\to\infty} \sum_{k=m}^{N} \binom{N}{k} \left(e^{-\frac{rT}{N}} up\right)^{N-k} \left(e^{-\frac{rT}{N}} d(1-p)\right)^k = N(d_1),$$

$$\lim_{N\to\infty} \sum_{k=m}^{N} \binom{N}{k} p^{N-k} (1-p)^k = N(d_2),$$

where $N(x) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$ is the cumulative standard normal distribution. d_1 and d_2 are given by

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)T\right], \quad d_2 = \frac{1}{\sigma\sqrt{T}} \left[\ln\frac{S}{K} + \left(r - \frac{\sigma^2}{2}\right)T\right].$$

Thus, $V_0 = \lim\limits_{N\to\infty} \left\{ e^{-rT} S_0 \sum_{k=m}^{N} \binom{N}{k} (up)^{N-k} \left(d(1-p)\right)^k - e^{-rT} K \sum_{k=m}^{N} \binom{N}{k} p^{N-k} (1-p)^k \right\}$

$$= S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2).$$

which is the same form as the Black-Scholes (B-S) formula of the European call option.

Following the same way, it can be proved that the limitation of the European put CRR model is the Black-Scholes (B-S) formula of the European put option.

APPENDIX B

ITO'S LEMMA

Definition Let (Ω, F, P) be a probability space. For each $\omega \in \Omega$, suppose there is a continuous function B_t of $t \geq 0$ that satisfies $B_0 = 0$ and that depends on ω. Then B_t, $t \geq 0$, is a Brownian motion if for all $0 = t_0 < t_1 < t_2 < \cdots < t_m$ the increments

$$B_{t_1} = B_{t_1} - B_0,\ B_{t_2} - B_{t_1},\ \ldots,\ B_{t_m} - B_{t_{m-1}}$$

are independent and each of these increments is normally distributed with

$$E\left[B_{t_{j+1}} - B_{t_j}\right] = 0,$$

$$Var\left[B_{t_{j+1}} - B_{t_j}\right] = t_{j+1} - t_j .$$

Definition Let $f(t)$ be a function defined for $0 \leq t \leq T$. The quadratic variation of f up to time T is

$$[f, f](T) = \lim_{\|\Pi\| \to 0} \sum_{j=0}^{n-1} \left[f(t_{j+1}) - f(t_j)\right]^2,$$

where $\prod = \{t_0, t_1, \cdots, t_n\}$ and $0 = t_0 \leq t_1 \leq \cdots \leq t_n = T$.

Theorem 1 Suppose that B_t is a Brownian motion, then

$$(dB_t)^2 = dt,\ dB_t dt = 0,\ (dt)^2 = 0.$$

Proof: Let $\prod = \{t_0, t_1, \cdots, t_n\}$ be a partition of $[0, T]$, then the sampled quadratic variation of the Brownian motion B_t is $Q_\Pi = \sum_{j=0}^{n-1}\left[B_{t_{j+1}} - B_{t}\right]^2$. By the definition of the Brownian motion,

$$E\left[\left(B_{t_{j+1}} - B_{t_j}\right)^2\right] = Var\left[\left(B_{t_{j+1}} - B_{t_j}\right)^2\right] - \left(E\left[B_{t_{j+1}} - B_{t_j}\right]\right)^2 = t_{j+1} - t_j \ ,$$

where $E\left[B_{t_{j+1}} - B_{t_j}\right] = 0$ and $Var\left[B_{t_{j+1}} - B_{t_j}\right] = t_{j+1} - t_j$. It implies that

$$EQ_\Pi = E\left[\sum_{j=0}^{n-1}\left[B_{t_{j+1}} - B_{t}\right]^2\right] = \sum_{j=0}^{n-1} E\left[B_{t_{j+1}} - B_{t}\right]^2 = \sum_{j=0}^{n-1}\left(t_{j+1} - t_j\right) = T \ .$$

Moreover,

$$Var\left[\left(B_{t_{j+1}} - B_{t_j}\right)^2\right] = E\left[\left(B_{t_{j+1}} - B_{t_j}\right)^4\right] - \left(E\left[\left(B_{t_{j+1}} - B_{t_j}\right)^2\right]\right)^2 = 3\left(t_{j+1} - t_j\right)^2 - \left(t_{j+1} - t_j\right)^2$$

$$= 2\left(t_{j+1} - t_j\right)^2.$$

Then,

$$Var\left[Q_\Pi\right] = Var\left[\sum_{j=0}^{n-1}\left[B_{t_{j+1}} - B_{t}\right]^2\right] = \sum_{j=0}^{n-1} Var\left[\left(B_{t_{j+1}} - B_{t}\right)^2\right] = \sum_{j=0}^{n-1} 2\left(t_{j+1} - t_j\right)^2$$

$$\le 2\sum_{j=0}^{n-1}\|\Pi\|\left(t_{j+1} - t_j\right) = 2\|\Pi\| T \to 0 \text{ as } \|\Pi\| \to 0 \text{ or } n \to \infty .$$

Thus, $\lim_{\|\Pi\| \to 0} Q_\Pi = T$.

Since $\int_0^T \left(dB_t\right)^2 = \lim_{\|\Pi\| \to 0}\sum_{j=0}^{n-1}\left(t_{j+1} - t_j\right) = T = \int_0^T dt$, hence $\left(dB_t\right)^2 = dt$.

Similarly, since

$$\left|\sum_{j=0}^{n-1}\left(B_{t_{j+1}} - B_{t}\right)\left(t_{j+1} - t_j\right)\right| \le \sum_{j=0}^{n-1}\left|\left(B_{t_{j+1}} - B_{t}\right)\left(t_{j+1} - t_j\right)\right| \le \sum_{j=0}^{n-1} \max_{0 \le k \le n-1}\left|B_{t_{k+1}} - B_{t_k}\right|\left(t_{j+1} - t_j\right)$$

$$= \max_{0\le k\le n-1}\left|B_{t_{k+1}}-B_{t_k}\right|\sum_{j=0}^{n-1}\left(t_{j+1}-t_j\right)=\max_{0\le k\le n-1}\left|B_{t_{k+1}}-B_{t_k}\right|\cdot T\ .$$

Because of the continuity of the Brownian motion, $\lim\limits_{\|\Pi\|\to 0}\max\limits_{0\le k\le n-1}\left|B_{t_{k+1}}-B_{t_k}\right|=0$, so that

$\lim\limits_{\|\Pi\|}\sum\limits_{j=0}^{n-1}\left(B_{t_{j+1}}-B_{t}\right)\left(t_{j+1}-t_j\right)=0$.

Furthermore, $\int_0^T dB_t dt=\lim\limits_{\|\Pi\|\to 0}\sum\limits_{j=0}^{n-1}\left(B_{t_{j+1}}-B_t\right)\left(t_{j+1}-t_j\right)=0$. Hence, $dB_t dt=0$.

Because of

$$0\le\sum_{j=0}^{n-1}\left(t_{j+1}-t_j\right)^2\le\sum_{j=0}^{n-1}\max_{0\le k\le n-1}\left|t_{k+1}-t_k\right|\left(t_{j+1}-t_j\right)=\max_{0\le k\le n-1}\left|t_{k+1}-t_k\right|\sum_{j=0}^{n-1}\left(t_{j+1}-t_j\right)$$

$$=\max_{0\le k\le n-1}\left|t_{k+1}-t_k\right|\cdot T=\left\|\Pi\right\|\cdot T\to 0$$

as $\left\|\Pi\right\|\to 0$.

Since $\int_0^T\left(dt\right)^2=\lim\limits_{\|\Pi\|\to 0}\sum\limits_{j=0}^{n-1}\left(t_{j+1}-t_j\right)^2=0$, hence $\left(dt\right)^2=0$.

Ito's lemma is known as Ito-Doeblin Theorem (Shreve, 2003), which is named for its discoverer - Japanese mathematician Kiyoshi Ito (September 7, 1951 - November 10, 2008). His work created a field of mathematics that is the calculus of stochastic variables.

Ito's lemma (for Brownian motion) Let $f\left(t,x\right)$ be a function for which the partial derivatives $f_t\left(t,x\right)$, $f_x\left(t,x\right)$ and $f_{xx}\left(t,x\right)$ are defined and continuous, and let B_t be a Brownian motion. Then, for every $T\ge 0$,

$$df\left(t,B_t\right)=f_t\left(t,B_t\right)dt+f_x\left(t,B_t\right)dB_t+\frac{1}{2}f_{xx}\left(t,B_t\right)dt\ .$$

Proof: Fix $T > 0$, and let $\Pi = \{t_0, t_1, \cdots, t_n\}$ be a partition of $[0, T]$, i.e., $0 = t_0 \le t_1 \le \cdots \le t_n = T$. Based on the Taylor's Theorem, the function $f(t, x)$ of both the time variable t and the variable x can be written as

$$f(t_{j+1}, x_{j+1}) - f(t_j, x_j)$$

$$= f_t(t_j, x_j)(t_{j+1} - t_j) + f_x(t_j, x_j)(x_{j+1} - x_j) + \frac{1}{2} f_{xx}(t_j, x_j)(x_{j+1} - x_j)^2$$

$$+ f_{tx}(t_j, x_j)(t_{j+1} - t_j)(x_{j+1} - x_j) + \frac{1}{2} f_{tt}(t_j, x_j)(t_{j+1} - t_j)^2 + \text{ higher-order terms.}$$

Replace x_j by B_{t_j}, replace x_{j+1} by $B_{t_{j+1}}$, then the sum

$$f(T, B_T) - f(0, B_0) = \sum_{j=0}^{n-1} \left[f(t_{j+1}, B_{t_{j+1}}) - f(t_j, B_{t_j}) \right]$$

$$= \sum_{j=0}^{n-1} f_t(t_j, B_{t_j})(t_{j+1} - t_j) + \sum_{j=0}^{n-1} f_x(t_j, B_{t_j})(B_{t_{j+1}} - B_{t_j}) + \frac{1}{2} \sum_{j=0}^{n-1} f_{xx}(t_j, B_{t_j})(B_{t_{j+1}} - B_{t_j})^2$$

$$+ \sum_{j=0}^{n-1} f_{tx}(t_j, B_{t_j})(t_{j+1} - t_j)(B_{t_{j+1}} - B_{t_j}) + \frac{1}{2} f_{tt}(t_j, B_{t_j})(t_{j+1} - t_j)^2 + \text{ higher-order terms.}$$

Take the limit as $\|\Pi\| \to 0$, then the first term on the right-side of the above equation contributes the ordinary (Lebesgue) integral

$$\lim_{\|\Pi\| \to 0} \sum_{j=0}^{n-1} f_t(t_j, B_{t_j})(t_{j+1} - t_j) = \int_0^T f_t(t, B_t) dt .$$

The second term contributes the Ito integral

$$\lim_{\|\Pi\| \to 0} \sum_{j=0}^{n-1} f_x(t_j, B_{t_j})(B_{t_{j+1}} - B_{t_j}) = \int_0^T f_x(t, B_t) dB_t .$$

The third term contributes another ordinary (Lebesgue) integral

$$\lim_{\|\Pi\| \to 0} \frac{1}{2} \sum_{j=0}^{n-1} f_{xx}(t_j, B_{t_j})(B_{t_{j+1}} - B_{t_j})^2 = \frac{1}{2} \int_0^T f_{xx}(t, B_t) dt .$$

The forth term can be observed that

$$\lim_{\|\Pi\|\to 0}\left|\sum_{j=0}^{n-1} f_{tx}\left(t_j, B_{t_j}\right)\left(t_{j+1}-t_j\right)\left(B_{t_{j+1}}-B_{t_j}\right)\right| \le \lim_{\|\Pi\|\to 0}\sum_{j=0}^{n-1}\left|f_{tx}\left(t_j, B_{t_j}\right)\left(t_{j+1}-t_j\right)\left(B_{t_{j+1}}-B_{t_j}\right)\right|$$

$$= \lim_{\|\Pi\|\to 0}\sum_{j=0}^{n-1}\left|f_{tx}\left(t_j, B_{t_j}\right)\right|\cdot\left(t_{j+1}-t_j\right)\cdot\left|B_{t_{j+1}}-B_{t_j}\right|$$

$$\le \lim_{\|\Pi\|\to 0}\max_{0\le k\le n-1}\left|B_{t_{k+1}}-B_{t_k}\right|\cdot\lim_{\|\Pi\|\to 0}\sum_{j=0}^{n-1}\left|f_{tx}\left(t_j, B_{t_j}\right)\right|\cdot\left(t_{j+1}-t_j\right)$$

$$= 0\cdot\int_0^T\left|f_{tx}\left(t, B_t\right)\right|dt = 0 \ .$$

The fifth term can be treated similarly,

$$\lim_{\|\Pi\|\to 0}\left|\sum_{j=0}^{n-1}\frac{1}{2} f_{tt}\left(t_j, B_{t_j}\right)\left(t_{j+1}-t_j\right)^2\right| \le \lim_{\|\Pi\|\to 0}\frac{1}{2}\sum_{j=0}^{n-1}\left|f_{tt}\left(t_j, B_{t_j}\right)\right|\left(t_{j+1}-t_j\right)^2$$

$$\le \frac{1}{2}\lim_{\|\Pi\|\to 0}\max_{0\le k\le n-1}\left|t_{k+1}-t_k\right|\cdot\lim_{\|\Pi\|\to 0}\sum_{j=0}^{n-1}\left|f_{tt}\left(t_j, B_{t_j}\right)\right|\cdot\left(t_{j+1}-t_j\right)$$

$$= \frac{1}{2}\cdot 0\cdot\int_0^T\left|f_{tt}\left(t, B_t\right)\right|dt = 0 \ .$$

The higher-order terms likewise contribute zero to the final answer.

Hence, the integral form is

$$f\left(T, B_T\right) = f\left(0, B_0\right) + \int_0^T f_t\left(t, B_t\right)dt + \int_0^T f_x\left(t, B_t\right)dB_t + \frac{1}{2}\int_0^T f_{xx}\left(t, B_t\right)dt \ .$$

Its differential form is

$$df\left(t, B_t\right) = f_t\left(t, B_t\right)dt + f_x\left(t, B_t\right)dB_t + \frac{1}{2}f_{xx}\left(t, B_t\right)dt \ .$$

Ito's lemma (for Ito's process) Let X_t, $t \ge 0$, be an Ito's process, and let $f\left(t, x\right)$ be a function for which the partial derivatives $f_t\left(t, x\right)$, $f_x\left(t, x\right)$ and

$f_{xx}(t,x)$ are defined and continuous, and let B_t be a Brownian motion. Then, for every $T \geq 0$,

$$df(t,X_t) = f_t(t,X_t)dt + f_x(t,X_t)dX_t + \frac{1}{2}f_{xx}(t,X_t)dX_t dX_t \ .$$

Proof: Proceed as in the sketch of the proof of Ito's lemma for Brownian motion, but with the Ito process X_t replacing the Brownian motion B_t.

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