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ON APPROXIMATING THE RUIN PROBABILITY AND THE MINIMUM INITIAL CAPITAL OF THE FINITE-TIME RISK PROCESS BY SEPARATED CLAIM TECHNIQUE OF MOTOR INSURANCE

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Abstract

In this paper, we study the minimum initial capital problem of an insurance company which has to hold the initial capital for ensuring that the ruin probability was not greater than the given quantity in the discrete-time risk processes. We separate claim severities into standard and large claim severities and consider the insolvency of the company

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with the ruin probability which is approximated by a simulation approach. Finally, the minimum initial capital is computed by the regression analysis.

1. Introduction

In the classical risk process, we consider claim severity Y_n occurred at the times T_n such that $0 \leq T_1 \leq T_2 \cdots \leq \cdots$. Thus the probability of insolvency (ruin) is only occurred at claim time T_n, $n \in \mathbb{N}$. The discrete time risk process is defined by

$$U_n = u + cT_n - \sum_{k=1}^{n} Y_k, \quad U_0 = u; \quad n = 1, 2, 3, \ldots, \tag{1}$$

where $u \geq 0$ is an initial capital, $c > 0$ is a premium rate for one unit time. In 2006, Chan and Zhang considered the discrete time risk process under the assumption $T_n = n$. Then the risk process (1) becomes

$$U_n = u + cn - \sum_{k=1}^{n} Y_k, \quad U_0 = u; \quad n = 1, 2, 3, \ldots, \tag{2}$$

where $\{Y_n, n \in \mathbb{N}\}$ is an independent and identically distributed (i.i.d.) claim severity process. The recursive and explicit formulas of the ruin probabilities were proposed with exponential and geometric claim severities. In 2013, Sattayatham et al. generalized the recursive formula of the ruin probability and introduced the minimum initial problems, controlling the ruin probability not exceed a given quantity.

In this paper, we assume that an insurance company is allowed to invest in a risk-free asset with a constant interest rate r for one unit time. Research models of this paper are two discrete time risk processes. The first process is given by

$$S_n = S_{n-1}(1 + r) + c - Y_n, \quad U_0 = u; \quad n = 1, 2, 3, \ldots, \tag{3}$$

where $u \geq 0$ is an initial capital, $c > 0$ is a premium rate for one unit time and $\{Y_n, n \in \mathbb{N}\}$ is an i.i.d. claim severity process. Moreover, in the process (3), the claim severity Y_n is separated into two types of claims, standard claim severities (V_n) and large claim severities (W_n). The criteria of separation is the mean (average) of claim severities. We shall construct the second process (eq. (4)) under the assumption that the standard claim and large claim are not occurred at the same time. We let $Z_n = T_n^l - T_{n-1}^l$, where T_n^l is the arrival time of nth large claim. Therefore, the second process is given by

$$U_n = \begin{cases} u, \; n = 0 \\ U_{n-1}(1+r) + c - V_n, \; n \neq \sum_{i=1}^{k} Z_i, \; \text{for all } k = 1, 2, 3, \ldots \\ U_{n-1}(1+r) + c - W_n, \; n = \sum_{i=1}^{k} Z_i, \; \text{for some } k = 1, 2, 3, \ldots. \end{cases} \tag{4}$$

Throughout this paper, we assume that all of processes, $\{Y_n, n \in \mathbb{N}\}$, $\{V_n, n \in \mathbb{N}\}$, $\{W_n, n \in \mathbb{N}\}$ and $\{Z_n, n \in \mathbb{N}\}$ are assumed to be i.i.d. and mutually independent. Next, we define the ruin probability of the discrete time risk processes (3) and (4), the probability of event that the surplus ever falls below zero before the time N, by

$$\Psi_N(u) = P(S_k < 0 \text{ for some } k = 1, 2, 3, \ldots, N \,|\, S(0) = u)$$

and

$$\Phi_N(u) = P(U_k < 0 \text{ for some } k = 1, 2, 3, \ldots, N \,|\, U(0) = u),$$

respectively.

2. Main Results

2.1. Data

We consider the data set of motor insurance claims for the year 2009 of an insurance company in Thailand; all types of vehicle, i.e., automobiles, lorries and motorcycles are included. We choose the considered data from a type of this company such that its claims are occurred in every day. These data are separated into the two kinds; standard and large claim severities by using the mean criteria. We show standard and large claim severities in Figure 1.

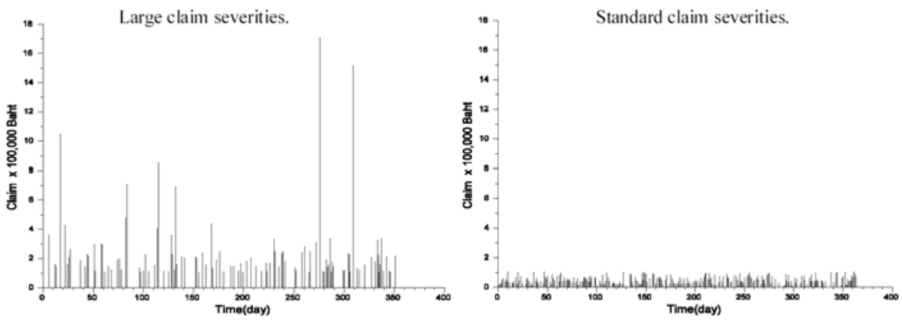

Figure 1

2.2. Parameter estimation

We compare the chi-squared value of the following distributions; log-normal, log-logistic, Burr, Weibull, exponential, gamma and Pareto distribution for the non-separated claim, the standard claim and large claim. Using the maximum likelihood estimator (MLE), we found that the non-separated claim accepts the lognormal distribution (2P), standard claim accepts Weibull distribution (2P) and large claim also accepts Weibull distribution (3P) with the smallest chi-squared value at 95% confidence. Next, the parameters of chosen distributions are improved by minimizing the chi-squared value with a randomized neighborhood search (RNS) approach. The results are shown in Table 2.

Table 2. Parameter estimations

Data	Probability density Function $f(x)$	MLE	Chi-squared value	RNS	Chi-squared value
Non-separated claim: Log-normal distribution (2P)	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}$	$\mu = 11.0630$ $\sigma = 0.9939$	5.8476	$\mu = 11.0579$ $\sigma = 0.9506$	1.1528
Standard claim: Weibull distribution (2P)	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$\alpha = 1.8864$ $\beta = 55085.00$	7.1221	$\alpha = 1.7548$ $\beta = 55178.61$	5.0267
Large claim: Weibull distribution (3P)	$\frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}$	$\alpha = 0.7548$ $\beta = 101150.00$ $\gamma = 105163.86$	7.6484	$\alpha = 7.6484$ $\beta = 101131.80$ $\gamma = 105163.86$	5.2890

In this research, we assume that Z_k is Poisson distribution and obtain the estimated parameter $\lambda = 3.4187$ by the MLE.

2.3. Simulation results

The simulation results are carried with 10,000 paths for the surplus processes (3) and (4) when $N = 365$ and initial capital $u = 0,$ 20,000, 40,000, …, 1,320,000 Baht. Figure 3 and Figure 4 explain the relation between ruin probabilities and the initial capitals; the top curves of these figures show the relation in case $\theta = 0.1,$ the next curves show the relation in case $\theta = 0.2$ and so on to the bottom curves show the relation in case $\theta = 1.0,$ respectively.

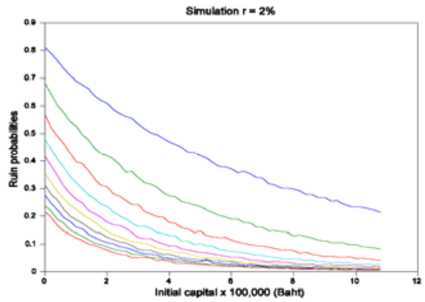

Figure 3. Ruin probability and initial capital in case of $r = 2\%$ (non-separated claim).

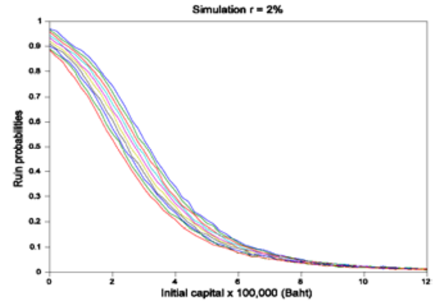

Figure 4. Ruin probability and initial capital in case of $r = 2\%$ (separated claim).

From Figures 3 and 4, we consider the relation of ruin probability $\Phi(u, 365)$ and initial capital u as an exponential function,

$$\Phi(u, 365) = \gamma \exp(-\delta u), \tag{5}$$

when γ and δ are approximated by exponential regression. Finally, we set the maximum acceptable risk with corresponding to the risk α. Therefore, under the regulation that the ruin probability has to be not greater than α, the initial capital has to satisfy the inequality, $u \geq -\frac{1}{\delta} \ln\left(\frac{\alpha}{\gamma}\right)$.

In case of non-dangerous portfolio or the premium rate is highly enough, the initial capital u may be negative. This means that this portfolio is not necessary to have the initial capital. Therefore, the minimum initial capital (MIC) is given by

$$MIC = \max\left\{0, \frac{\ln \gamma - \ln \alpha}{\delta}\right\}. \tag{6}$$

Table 5 shows the insurance company has to hold the MIC for ensuring the ruin probability is not greater than $\alpha = 0.01$ with interest rate $r = 2\%$.

Table 5. MIC (Baht) in case $r = 2\%$, $\alpha = 0.01$

Safety loading θ	Non-separated claim		Separated claim	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	109,647.98	3,607,340	122,551.64	1,253,549
0.2	119,615.98	2,168,608	133,692.70	1,251,732
0.3	129,583.98	1,649,868	144,833.75	1,233,700
0.4	139,551.98	1,371,444	155,974.81	1,221,933
0.5	149,519.98	1,213,687	167,115.87	1,222,011
0.6	159,487.98	1,109,112	178,256.93	1,211,145
0.7	169,455.97	1,014,798	189,397.99	1,193,560
0.8	179,423.97	936,609	200,539.04	1,193,374
0.9	189,391.97	877,268	211,680.10	1,167,647
1.0	199,359.97	827,780	222,821.16	1,168,583

The premium rates c in Table 5 are computed by the expected value premium principle, i.e., $c = (1 + \theta)E[Y_1]$ and $c = (1 + \theta)\left(\frac{EW_1}{EZ_1} + EV_1\right)$ in the case of non-separated and separated claims, respectively.

From Table 5, we perform linear regression analysis between the premium rate and the MIC. The results are shown in Figure 6.

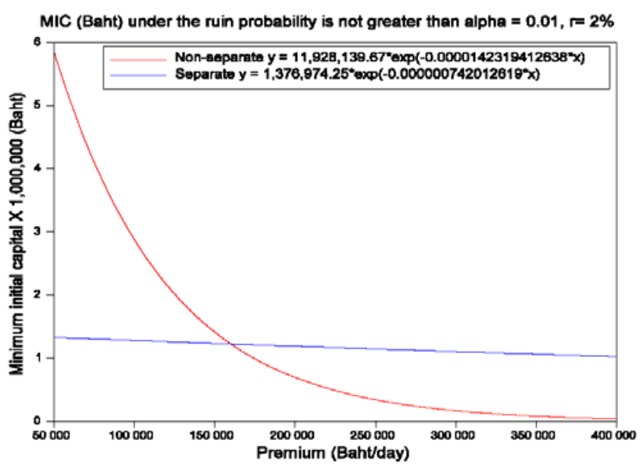

Figure 6. MIC with the premium rate $\alpha = 0.01$, $r = 2\%$.

3. Discussions

From Figure 6, we can conclude that the separated claim method is better than non-separated claim method in the view of the sensitivity analysis. Nevertheless, in the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$MIC = \begin{cases} 11{,}928{,}139.67 \exp(-0.0000142319412638x), & \text{if } x \le 160{,}046 \\ 1{,}376{,}974.25 \exp(-0.000000742012619x), & \text{if } x > 160{,}046, \end{cases}$$

where $x = 160{,}046$ is the intersection point of two curves.

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