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Dynamic Risk Measurement of Financial Time Series with Heavy-Tailed: A New Hybrid Approach

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Abstract : This paper proposes a new hybrid approach to measure dynamic risk of financial time series with heavy-tailed distribution. The proposed method, hereafter referred to as NIG-MSA, exploits the normal inverse Gaussian (NIG) distribution to fit the heavy-tailed distribution, and employs the empirical mode decomposition to structure a multi-scale analysis (MSA) methodology. The validity of NIG-MSA method for volatility prediction is confirmed through Monte Carlo simulation. This method is illustrated with an application to the risk measurement of returns on $S\&P500$ index and our results show that the proposed NIG-MSA approach provides more precise value at risk calculation than the traditional single-scale model.

Keywords : value at risk; dynamic quantile; empirical mode decomposition; heavy-tailed distribution.

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1 Introduction

Over recent years, financial markets have become much more volatile compared to previous decades. The most difficult task in the analysis of financial markets is to measure financial risks accurately. Financial risk measurement has been addressed by an increasing number of researches (Szego, 2002[1]; Tsukahara, 2014[2]). Financial risks have many sources and are typically mapped into a stochastic framework with various kinds of risk measures such as value at risk (VaR), Conditional Value at Risk (CVaR), and spectral risk measures. Among them, VaR has become the most frequently used risk measure. In many practical applications, VaR at τ probability level is commonly defined as:

$$VaR_{\tau,t} = F_t^{-1}(\tau) = \sigma_t q_t, \tag{1.1}$$

where F_t^{-1} is the inverse function of the conditional cumulative Gaussian distribution function of the underlying at time t (Franke et al., 2004[3]). q_τ denotes the $\tau - th$ quantile of the distribution of innovation term ε_t, i.e., $P(\varepsilon_t < q_\tau) = \tau$, and σ_t denotes the volatility. Since the VaR can be expressed as $\sigma_t \times q_\tau$, so it is crucial for the calculation of VaR to model the distribution of the innovation term and estimate the volatility accurately.

It is clear that the accuracy of VaR depends heavily on the assumption of the underlying distribution, which often assumed that the involved risk factors are normally distributed for reasons of stochastic and numerical simplicity. However, many empirical studies have shown that the financial returns have leptokurtic distribution with high peak and fat tails (Peiro, 1999[4]; Verhoeven, 2004[5]). The distribution assumption of the innovative term influences the performance of the VaR. The models based on the normality assumption achieve almost the same values at the 5% quantile as those with a leptokurtic distribution. However, if one considered lower quantiles such as 1% quantile, the normality assumption becomes invalid, because the difference relative to the normal becomes larger for lower quantiles. The NIG distribution is a heavy-tailed distribution that can well replicate the empirical distribution of the financial risk factors (Y. Chen et al., 2008[6]). In this paper, we discuss the application of this distribution in financial risk measurement.

Accurate volatility modeling is in the focus of the financial econometrics and quantitative finance research. The most commonly used volatility models is generalized autoregressive conditional heteroscedasticity (GARCH) models proposed by Bollerslev (1992)[7], who extend the seminal ideas of Engle (1982)[8] about ARCH models. Their prominent popularity stems from their ability to formulate conditional variance of returns. To improve the limitation of the GARCH model, leverage effects and long memory effects, many extended models were proposed Babsiri and Zakoian, 2001[9]. Many empirical findings suggest that GARCH models are able to capture volatility persistence, clustering or asymmetry (Bentes, 2015[10]). However, it is well known that financial time series is inherently nonstationary (Guhathakurta et al., 2008[11]). While the GARCH models and their extensions were developed for stationary processes, which usually neglect the fact that the form of the volatility model is time-unstable. Therefore, one must employ a new model designed genuinely for the non-stationary financial data to measure the financial risk.

Financial data is usually not constant or absolute scale and usually with multiple time-scale characteristics (Skjeltorp, 2000[12]). So it becomes important for us to take a multi-scale analysis for financial data (Guhathakurta et al., 2008[11]; Huang et al, 2003[13]). Multi-scale analysis is a comprehensive analysis approach and specially developed for non-stationary processes. It has been widely used in the fields of industrial engineering and signal processing. In general, the multi-scale analysis consists of two steps: (1) Decompose the original signal according to the time scale and (2) integrate the analysis results of subsystems. In this paper, we adopt empirical mode decomposition (EMD) method for decompose process, and averaging method for integrate process. The EMD method proposed by Huang (1998)[14] can adaptively decompose the original signal into a series of intrinsic mode function components with different time-scale. The method is applicable to nonlinear and non-stationary processes since it is based on the local characteristic time scale of the data. Compared to wavelet decomposition and Fourier decomposition, EMD decomposition has been reported to have worked better in describing the local time scale. The EMD method has been applied to analyze the non-stationary financial time series (Huang et al., 2003[13]; Premanode et al., 2013[15]; Hong L, 2011[16]). For non-stationary financial time series, the multi-scale method has been addressed by an increasing number of researches. Most of the literature has concentrated on the prediction of crude oil price and stock index (Yu et al., 2008[17]). To the best of our knowledge, using the multi-scale method to forecast VaR of financial market has not been studied so far.

In this paper, we intend to improve the risk measurement model by following the steps: Firstly, we decomposed the financial time series into several intrinsic mode functions by the empirical mode decomposition. Secondly, the GARCH model is used to forecast the volatility of the each intrinsic mode function components respectively. Finally, the volatility that has been predicted before will be integrated by the averaging method. The NIG-MSA model proposed in this paper can be easily applied to the problem of VaR calculation. We believe that the contributions of this paper are:

(1) The"divide-and-conquer"strategy is proposed to predict volatility of return series, which consist of EMD decomposition and averaging integration.

(2) The NIG distribution is suggested to model the distribution of stochastic term in GARCH model, which can perfectly fit the devolatilized returns.

(3) Introducing the concept of time-varying quantile, which can be used to calculate the dynamic VaR of return process.

The remainder of the paper is structured as follows. Section 2 discusses the properties of the NIG distribution and describes the NIG-MSA dynamic risk measurement model. In section 3 the validity of the NIG-MSA technique is shown via comparing with other volatility prediction models. Using $S\&P500$ yield series, the performance of the NIG-MSA risk measurement model is presented by means of back-testing in section 4. Finally, Section 5 draws the conclusion and discussion.

2 The Dynamic Risk Measurement Model

In this section, the overall process of formulating the dynamic risk measurement model is presented. Here we name this new VaR method as Normal Inverse Gaussian- Multi-Scale Analysis (NIG-MSA) method. The NIG-MSA model ensemble paradigm can be formulated as illustrated in Figure 1.

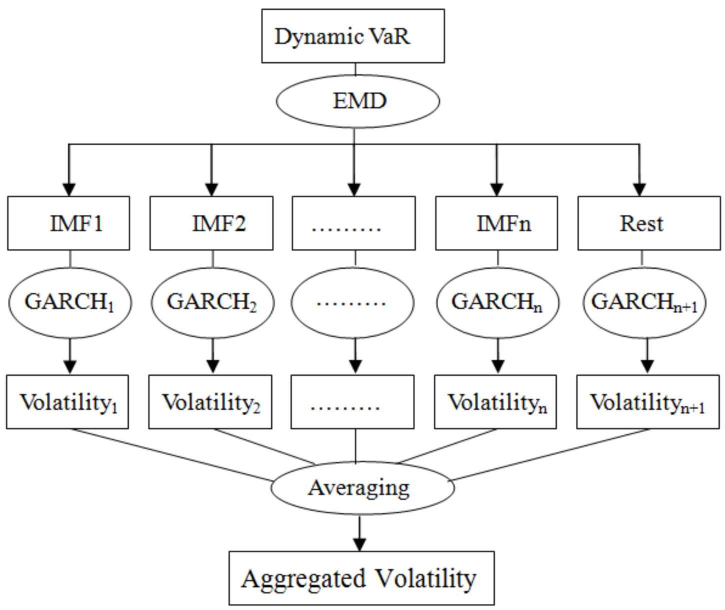


Figure 1: The overall process of the NIG-MSA model

As can be seen from Figure 1, the NIG-MSA model generally consists of the following four main steps:

(1) The returns series $R(t)$, $t = 1, 2, ..., T$ is adaptively decomposed into a finite number of IMF (Intrinsic Mode Function) components employed the EMD method.

(2) The GARCH $(1, 1)$ model is used as a prediction tool to model the volatility process of each extracted IMF component and to predict the corresponding volatility, in which we assume the innovation term is NIG distribution.

(3) The volatility forecasting results of all extracted IMF components in step (2) are integrated to generate an aggregated volatility estimation using an averaging method.

(4) Using the aggregated volatility to calculate the devolatilized return, then the NIG distribution parameters can be estimated.

2.1 The GARCH $(1, 1)$ Model with NIG Distribution

The GARCH $(1, 1)$ model is a parsimonious model in volatility forecasting models (Eberlein, 2003[18]). The model provides a simple representation of the main statistical characteristics of a return process, such as autocorrelation and volatility clustering. The GARCH $(1, 1)$ model is the most popular structure for volatility forecasting and, consequently, it is extensively used to model real financial time series.

Let $R_t = logP_t - logP_{t-1}$ denote the logarithm of return, where P_t is the asset price at time t. The return process is modeled in the GARCH $(1, 1)$:

$$R_t = \sigma_t \varepsilon_t \tag{2.1}$$

$$\sigma_t^2 = \omega + \phi R_{t-1}^2 + \varphi \sigma_{t-1}^2, \tag{2.2}$$

where the innovation term ε_t is assumed to be an independently and identically distributed random variable. The volatility σ_t^2 is time varying and unobservable in the market. To ensure that the conditional variance is positive, we assume that the parameters ω, ϕ and φ all satisfy $\omega > 0, \phi, \varphi > 0$.

The NIG distribution is a heavy-tailed distribution which is rich enough to model financial time series and has the benefit of numerical tractability (Eberlein et al., 1995[19]). The density function of the NIG distribution for x is

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha\delta}{\pi} \cdot \frac{K\alpha\sqrt{\delta^2 + (x-\mu)^2}}{\sqrt{\delta^2 + (x-\mu)^2}} \exp\{\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu)\}, \tag{2.3}$$

where, $\delta > 0$ and $|\beta| \leq \alpha$, $K(x) = \frac{1}{2}\int_0^\infty \exp\{-\frac{x}{2}(y + y^{-1})\}dy$.

The location and scale of the density are mainly controlled by parameters μ and δ respectively, whereas α and β play roles in the skewness and kurtosis of the distribution. Thus all moments of $NIG(\alpha, \beta, \delta, \mu)$ have simple explicit expressions, in particular, the mean and variance are $E(x) = \mu + \beta\delta/\sqrt{\alpha^2 - \beta^2}$ and $VaR(x) = \alpha^2\delta/\sqrt{(\alpha^2 - \beta^2)^3}$. Furthermore, if $\mu = 0$, the NIG distribution has the tail-behaviors

$$f_{NIG}(x, \alpha, \beta, \delta, \mu = 0) \sim x^{-\frac{3}{2}} e^{-(\alpha-\beta)x}, \quad as \quad x \to \infty, \tag{2.4}$$

which shows that the NIG distribution has an exponential decaying speed. As compared to the normal distribution, the NIG distribution decays more slowly and the NIG distribution often appears in modeling the return process. In this paper, we propose that the stochastic term ε_t is assumed to possess the NIG distribution. The parameters in GARCH $(1, 1)$ model are estimated using quasi-maximum likelihood method.

2.2 Empirical Mode Decomposition (EMD)

The decomposition is based on the local characteristic time scale of the data. So any non-stationary dataset can be adaptively decomposed into a finite and often small number of Intrinsic Mode Functions (IMF) with individual intrinsic time scale properties. The IMF satisfies the following two prerequisites: (1) In the whole data series, the number of extreme points and the number of zero crossings must be equal or differ at most by one. (2) The mean value of the envelopes defined by local maxima and minima must be zero at all points. Each IMF component has a clear physical meaning and contains a certain characteristic range of time scale (Huang et al., 1998[14]). As compared with the original data, the IMF components are more stationary, which is advantageous to forecast volatility of return process. The generic EMD algorithm is described by the following steps:

i) Identify all the maximum points and all the minimum points of original signal $x(t)$.

ii) Fit the maxima envelope $x_u(t)$ and minima envelope $x_l(t)$ with cubic spline function.

iii) Calculate the mean value $m_1(t) = (x_l(t) + x_u(t))/2$.

iv) Calculate the quasi-IMF $h_1(t) = x(t) - m_1(t)$ and test whether $h_1(t)$ satisfies the two prerequisites of an IMF property. If they are satisfied, we obtain the first IMF. If not, we regard $h_1(t)$ as $x(t)$ and repeat steps (i)-(iii) until $h_1(t)$ becomes an IMF.

v) Calculate the first residual term $res(t) = x(t) - h_1(t)$. The $res(t)$ is treated as new input $x(t)$ in the next loop to derive the next IMF. We stop the decomposition procedure until the residual term $res(t)$ becomes a monotonic function from which no further IMF can be extracted.

From the above decomposition process, it is obvious that the original time series $x(t)$ can be reconstructed by summing up all the IMF components together with the last residue component, that is $x(t) = \sum h_i(t) + res(t)$. In this paper, the residual term is seen as the last IMF.

EMD method adaptively obtains the local IMF components with the shortest cycle by screening the local characteristics from the original signal and each component also includes a corresponding section of different frequency component.

3 Simulation Experiment

The NIG-MSA technique consists of two main parts: predict the volatility using the multi-scale methodology and dynamically estimate the quantile of innovation. The calculation procedure can be described as:

i) Set up the data generating model.

ii) Estimate the aggregated volatility $\hat{\sigma}_t$ using the multi-scale methodology.

iii) Calculate the innovation terms $\varepsilon_t = R_t/\hat{\sigma}_t$ and fit the NIG distribution parameters and estimate the quantile $\hat{q}_\tau$.

iv) Calculate the $VaR_t = \hat{\sigma}_t \cdot \hat{q}_\tau$.

From the above calculation steps, we can see that the key pillar for the NIG-MSA technique is the accurate estimation of the volatility. In the simulation, we only focus on the volatility forecasting. The Monte Carlo simulation is applied to evaluate the performance of the NIG-MSA method. The simulated data set is generated by the following model

$$R_t = \sigma_t \varepsilon_t \tag{3.1}$$

$$\sigma_t^2 = \begin{cases} 0.1 + 0.4R_{t-1}^2 + 0.5\sigma_{t-1}^2, 1 \leq t \leq 400; \\ 0.5 + 0.1R_{t-1}^2 + 0.8\sigma_{t-1}^2, 400 < t \leq 750; \\ 0.1 + 0.7R_{t-1}^2 + 0.2\sigma_{t-1}^2, 750 < t \leq 1000, \end{cases} \tag{3.2}$$

where R_t is return and ε_t is innovation distributed as normal inverse Gaussian with zero mean and unit variance. Notice that if the data generating process is $R_t = \sigma_t \varepsilon_t$ then $VaR_t = \sigma_t q_\tau$ (Franke et al., 2004[3]).

The purpose of this experiment is to evaluate the four volatility forecast methods: (i) GARCH $(1, 1)$ with normal distribution (Nor-GAR), (ii) GARCH $(1, 1)$ with normal inverse Gaussian distribution (NIG-GAR), (iii) Multi-scale analysis with normal distribution (Nor-MSA) and (iv) Multi-scale analysis with normal inverse Gaussian distribution (NIG-MSA). We use the four models to respectively forecast the real volatility generated in (3) and the simulation results are shown in Figure 2.

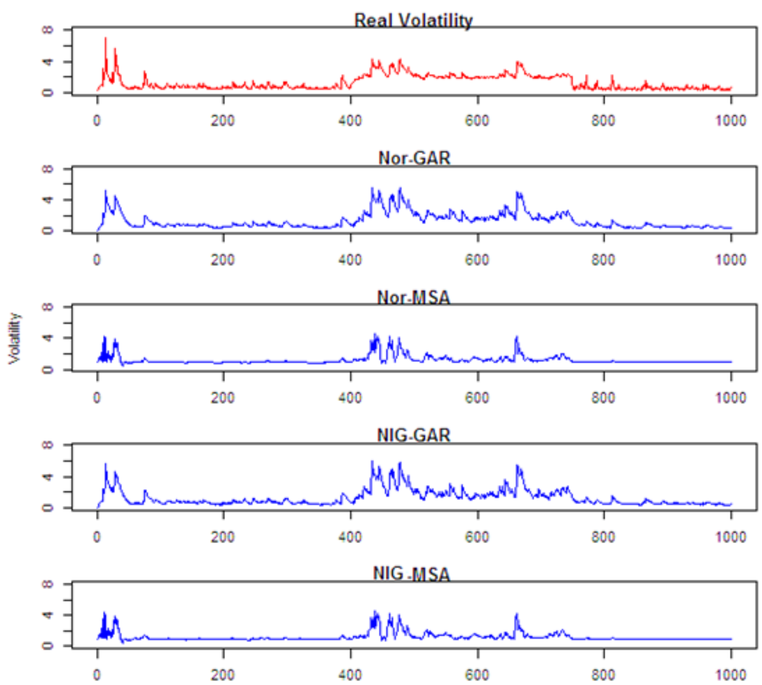


Figure 2: The comparison of volatility forecasting

The volatility forecasting performance is evaluated using the following statistical metrics.

Normalized mean squared error (NMSE):

$$NMSE = \sqrt{\sum_{t=1}^{N}(\hat{\sigma}_t^2 - R_t^2)^2 / \sum_{t=1}^{N}(R_{t-1}^2 - R_t^2)^2} \tag{3.3}$$

Normalized mean absolute error (NMAE):

$$NMAE = \sum_{t}^{N} |\hat{\sigma}_t^2 - R_t^2| / \sum |R_{t-1}^2 - R_t^2| \tag{3.4}$$

Hit rate (HR)

The three statistical metrics relate the predicted volatility $\hat{\sigma}_t^2$ to the proxy volatility estimation R_{t-1}^2. The NMSE and NMAE are the measures of the deviation between the proxy and predicted values. The smaller their values, the closer the predicted volatility is to the actual values. The HR is a measure of how often the model gives the correct direction of change of volatility. The larger the value of HR, the better is the performance of prediction.

Additionally, the volatility of the return process can not be observed, so we evaluate the performance of the volatility prediction in the model following the criterion: the better the forecasting performance of volatility model, the better the standardized observation ($\varepsilon_t = \hat{R}_t/\hat{\sigma}_t$) is fitting the normal inverse Gaussian distribution. The Kolmogorov- Smirnov distance (KS) is usually used to test whether a given $F(x)$ is the underlying probability distribution of $F_n(x)$, so we use the Kolmogorov-Smirnov distance as the criterion for the goodness of fit testing. It is defined as

$$KS = sup_{x\in R}|F(x) - F_n(x)|, \tag{3.5}$$

where $F(x)$ is the empirical sample distribution and $F_n(x)$ is the cumulative distribution function. The smaller the values of KS distances, the closer are the predicted volatility to the actual values.

Table 1 gives the descriptive statistics of the simulation results and shows that superiority of the NIG-MSA model over the other models. The table reports the computed KS distance, NMSE, NASE and HR statistics metrics for the four models: Nor-GAR, Nor-MSA, NIG-GARCH, NIG-MSA. It shows that the value of KS distance, NMSE, NMAE for the NIG-MSA model are below the others, and that the value of HR for NIG-MSA model is the highest. Further, it indicates that multi-scale analysis has more influence on the volatility forecasting performance compared with the NIG distribution assumption. For an example of KS distance, the value of Nor-MSA reduced to 0.0174 and NIG-GAR only down to 0.0402 relative to the value of Nor-GAR 0.0581. As for the other three statistical metrics, we can draw the same conclusion. The NIG-MSA model can give better predictions because of its good time-frequency property which can describe non-stationary financial time series.

4 Empirical Analysis

The data set $S\&P$ 500 index was used in our empirical analysis. The index is daily registered from 2000/01/03 to 2014/10/28. There are 3729 observations. The first 2768 observations (from 2000/01/03 to 2010/12/31) are used as a basis to train the multi-scale analysis system and estimate the NIG distribution parameters. The residual 961 observations are used as a test set to evaluate the prediction of the dynamic VaR calculated by the NIG-MSA dynamic risk measurement model. The graphics of the return processes of train set are displayed in Figure 3.

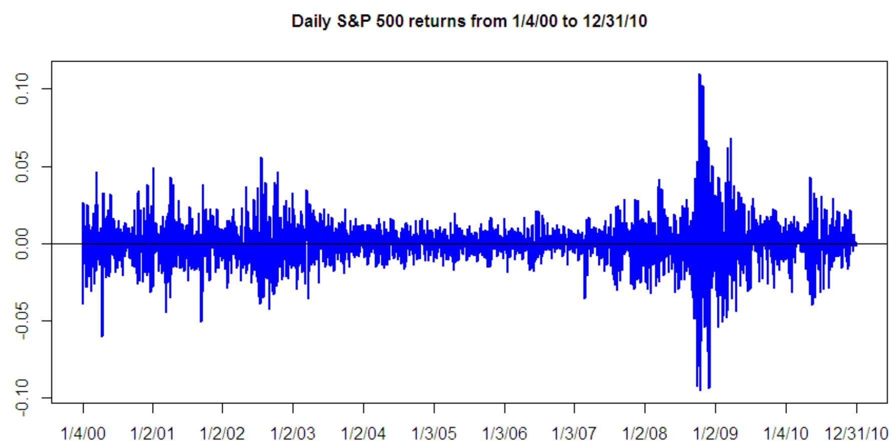


Figure 3: The logarithmic return process of $S\&P$ 500 index

4.1 NIG-MSA Model Training and Volatility Prediction

The NIG-MSA model can be trained according to the multi-scale methodology shown in section 2. Firstly, the training set (2768 observations) is decomposed into ten IMF components (the last IMF is residual term) using the EMD technique, as illustrated in Figure 4. Then, the GARCH $(1,1)$ model was used to model the every IMF component and to estimate corresponding volatility. The estimation results of the parameters as shown in Table 2. Finally, we use the averaging method to integrate the volatilities of IMF components and the aggregated volatility is given in Figure 5.

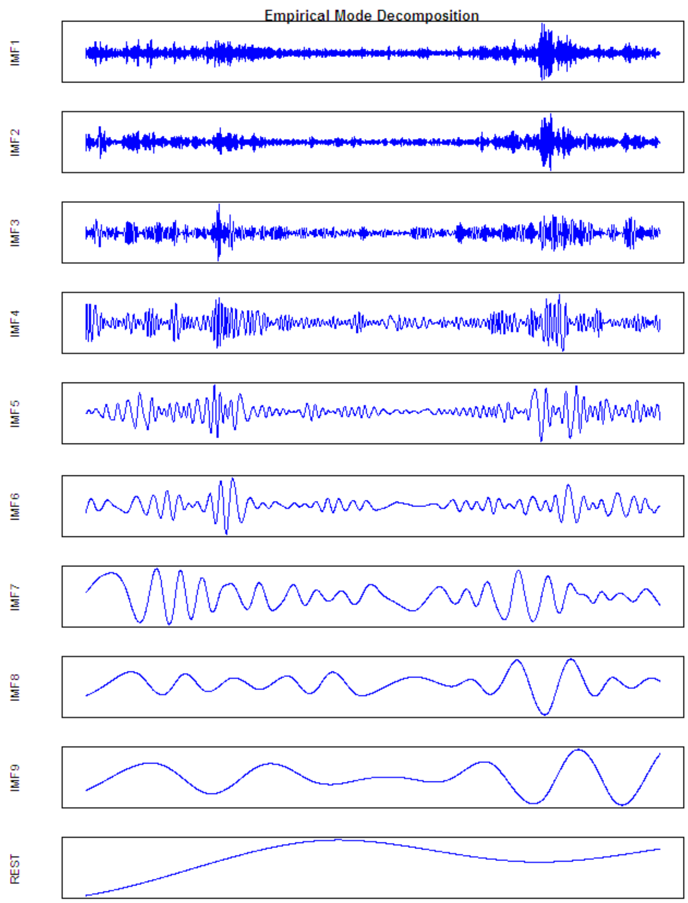


Figure 4: The decomposition of the training set

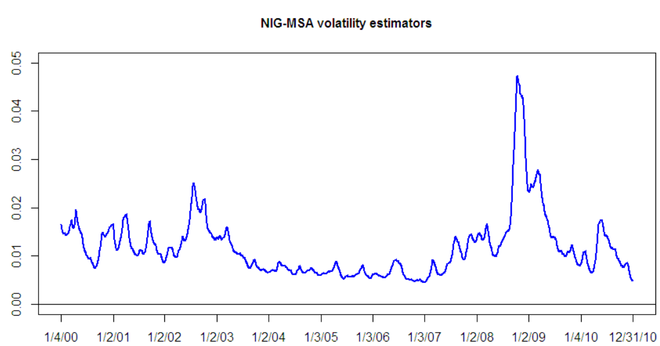


Figure 5: The volatility estimation of the training set

The NIG-MSA model that has been trained can be used to predict the volatility of the test set series. The basic idea of the volatility estimation comes from the assumption that although the returns series is non-stationary in a long time period, its volatility structure is relatively stationary. So, we suppose that the test set consists of 10 IMF components, and use the GARCH $(1,1)$ model which has been modeled to forecast the volatility of the each component. Then the 10 volatility prediction series are integrated to generate the final volatility prediction. In order to evaluate the performance of the NIG-MSA model, the Nor-GAR model is selected as the reference method, their prediction results of volatility as shown in Figure 6.

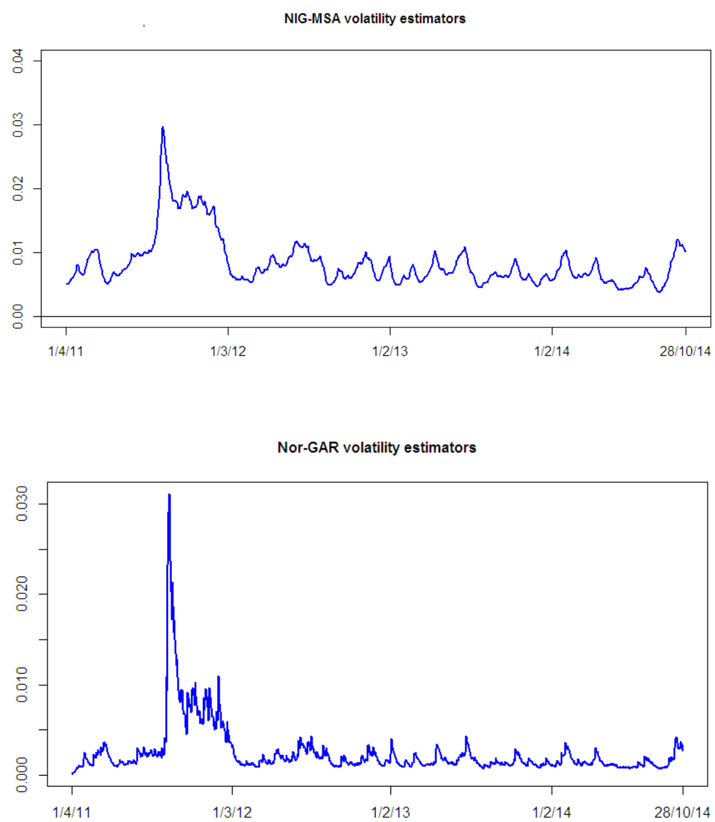
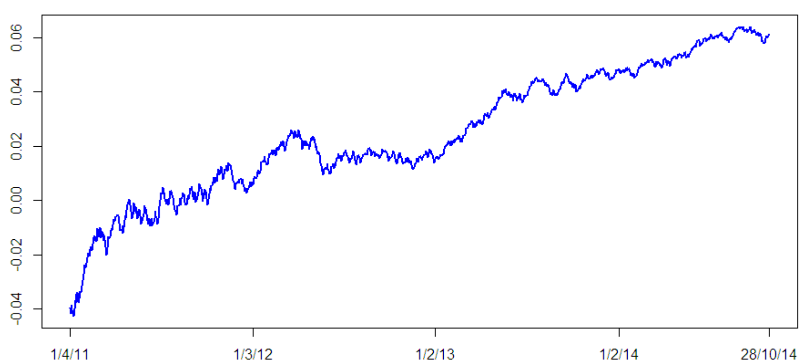


Figure 6: The volatility estimation of the test set

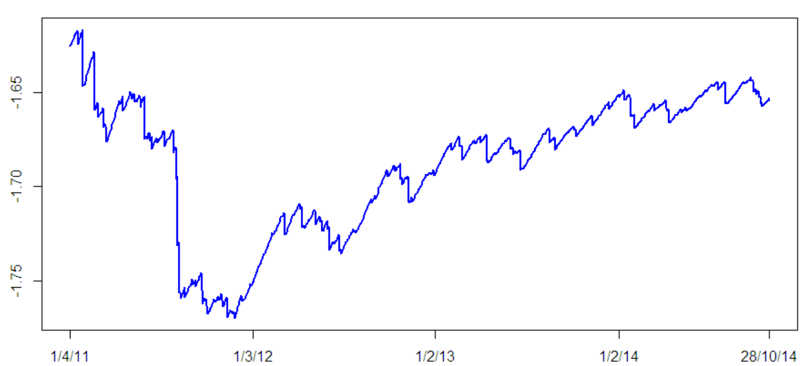
4.2 Time-Varying Quantile Estimation

In the NIG-MSA model, the distribution parameters could be time-variant as well. Figure 7 shows the quantile varies as time passes, which means that we could not keep the how assumption that the devolatilized returns are identically distributed. Instead, we estimated the dynamic quantiles based on the test set data. In Figure 7, we show the dynamic quantile estimations of the three probability levels, from the top the evolving NIG quantiles for $\tau = 0.5, \tau = 0.05$ and $\tau = 0.005$. A more detailed description is shown in Table 3 which contains four statistical metrics: Minimum, Maximum, Mean and Standard deviation. It gives the descriptive statistics of dynamic quantiles estimated by NIG-MSA technique. This provides evidence that the more extreme the probability levels, the greater the quantile varies as time passes. For the extreme probability $\tau = 0.005$, the variety range value is 0.5244 and the standard deviation is 0.1156. However, for the probability $\tau = 0.5$, the variety range value is 0.1064 and the standard deviation is 0.0236. This inspires us to consider that we should use dynamic quantiles to calculate the VaR, especially for the extreme events.

tau=0.5



tau=0.05



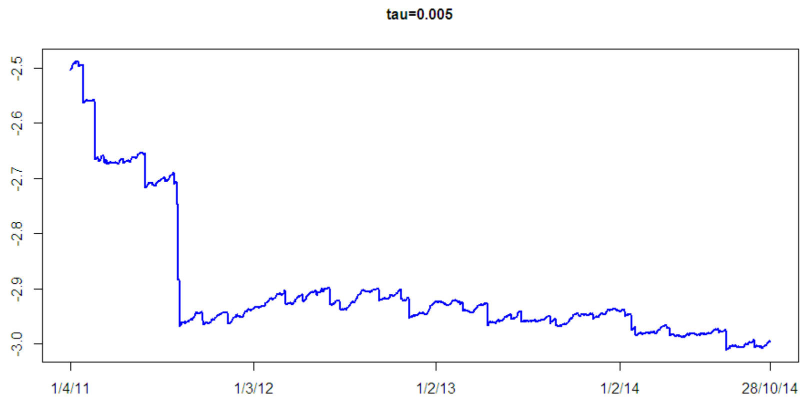


Figure 7: Dynamic quantiles estimation of test set

4.3 Value at Risk and Backtesting

Value at risk (VaR) can answers the question: How much can one lose with τ probability over the pre-set horizon. The volatility estimation as well as the distribution assumption of the devolatilized returns is essential to the VaR based risk management. We can calculate VaR using the formula $VaR_{\tau,t} = \sigma_t q_\tau$. But in practice, one is interested in the prediction of VaR. In the NIG-MSA approach, we robustly estimated the volatility $\hat{\sigma}_t$. Because the volatility process is a supermartingale, so we use the estimate today as the volatility forecast $\widetilde{\sigma}_{t+1}$ for tomorrow, i.e. $\widetilde{\sigma}_{t+1} = \hat{\sigma}_t$. Further, we calculate the NIG distribution parameters of the devolatilized returns and estimated the dynamic quantile $\hat{q}_\tau$. Then, the VaR at the probability level τ was predicted as

$$VaR_{\tau,t+1} = \hat{\sigma}_{t+1}\hat{q}_\tau. \tag{4.1}$$

The daily VaR predictions of $S\&P500$ returns test set are displayed in Figure 8. The VaR forecasts are different between the NIG-MSA model and the Nor-GAR model. At the 5% probability level, there are more than 45 exceptions observed in Nor-GAR model and more than 48 exceptions observed in NIG-MSA model. Their exception rate respectively is 4.47% and 4.99%, both are very close to the probability level 5%. However, at the 0.5% probability level, the exceptions rate of the two models is 0.68% and 0.44% respectively. This means that as the probability level decreases to some extreme level, the gaps of these two models get larger. Figure 8 gives the quantitative statistics of the testing set.

We employ the back testing procedures to evaluate the validation of the VaR calculation. The standard is that a VaR calculation should not underestimate the market risk. Let N denote the number of exceptions at time t, $t = 1, 2, ..., T$. We hope that the proportion of exceptions N/T equal with the fixed probability level

τ. The hypothesis test is given as:

$$H_0 : E[N] = T\tau, \qquad H_1 : E[N] \neq T\tau.$$

Jorion (2001) proposed using the likelihood ratio statistic

$$LR = -2log[(1-\tau)^{T-N}\tau^N] + 2log[(1-N/T)^{T-N}(N/T)^N], \tag{4.2}$$

to test this hypothesis. Under H_0, the statistic LR is asymptotically $\chi^2(1)$ distributed.

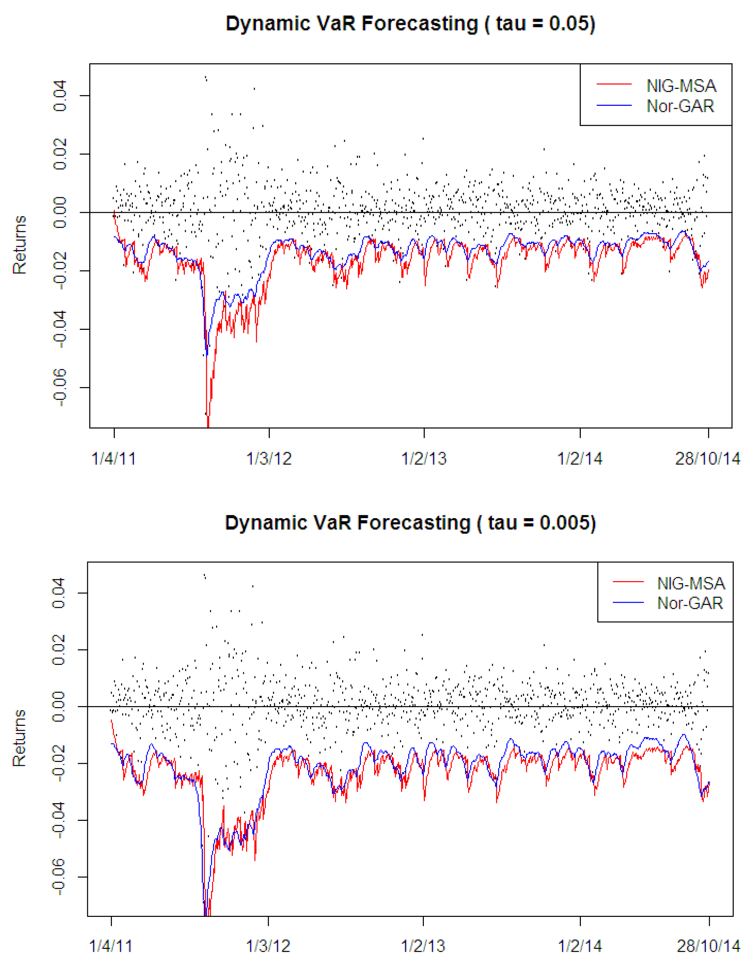


Figure 8: Dynamic Value at Risk forecasting of test set

Table 4 summarizes the results of the backtesting for the test data. We compare the NIG-MSA model with the Nor-GAR model under four probability levels:

0.5%, 1%, 2.5% and 5%. It shows that the NIG-MSA model gives more accurate predictions at each probability level than the Nor-GAR model. Especially under extreme probability level 1% and 0.5%, the Nor-GAR model fails to provide acceptable results under 95% confidence level.

5 Conclusion and Discussion

This study proposes the NIG-MSA risk measurement model based on the multi-scale volatility estimation and the normal inverse Gaussian distribution. Since most of the financial data are inherently non-stationary and the distribution of the devolatilized returns is leptokurtic and asymmetric, it is important that we adopt an approach designed for such characteristics. The multiple time-scale analysis method is specially developed for analyzing non-stationary financial time series. The volatility estimation based on decomposition and integration will be more accurate and robust. The distribution of the innovations can be perfectly modeled by the NIG distribution. So we propose the NIG-MSA model to measure the financial risk metrics.

The NIG-MSA model is proposed firstly, which mainly includes three key techniques: EMD decomposition, GARCH model and averaging integration. Then we compared the volatility forecasting performance of the NIG-MSA model with the other three models (Nor-GAR, NIG-GAR and Nor-MSA) using a simulated data sets. As demonstrated in the experiment, the VaR forecasts calculated by NIG-MSA technique significantly better than Nor-GAR model in all aspects. The superior performance of NIG-MSA model to the Nor-GAR model mostly lie in that the NIG-MSA fully considering the non-stationary feature of financial time series and the non-normal distribution of the devolatilized returns. The proposed method can be applied to predict dynamic risk measurement.

For the future research, we will extend the technique of multi-scale to analysis the non-stationary financial time series. On the one hand, we expect to improve the performance of multi-scale analysis system, such as the approach selection of decomposition and integration. On the other hand, it is more interesting and challenging to measure the risk metrics of portfolio in financial markets. We expect to apply the idea of multiple time-scale analysis to forecast volatility with the multivariate NIG distribution.

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